Large-eddy simulation of bubbly turbulent flows based on an Euler-Lagrange approach for a huge number of microbubbles

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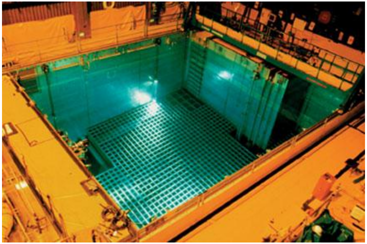
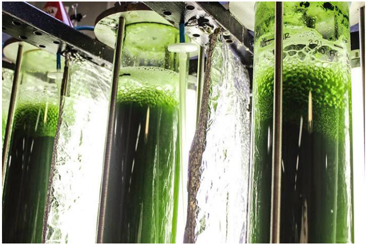
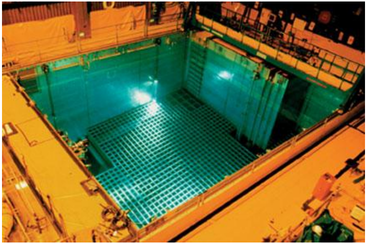
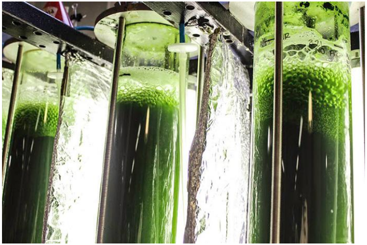
HELMUT SCHMIDT UNIVERSITÄT

14th Workshop
on Two-Phase Flow Predictions
Halle (Saale), Sept. 07.-10. 2015







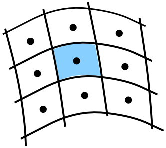


Accurate simulation of turbulent bubbly flows at high volume loadings

LESOCC

Large **E**ddy **S**imulation **O**n **C**urvilinear **C**oordinates

- Navier–Stokes solver (incompressible fluid)



- 3–D finite-volume approach
  - Curvilinear body–fitted coordinate system
  - Non–staggered (cell–centered) grid arrangement
  - Block–structured grids

- Spatial discretization
  - Viscous fluxes: central differences $\mathcal{O}(\Delta x^2)$
  - Convective fluxes: five different schemes, central diff. $\mathcal{O}(\Delta x^2)$, **CDS–2**

LESOCC

Large **E**ddy **S**imulation **O**n **C**urvilinear **C**oordinates

- Temporal discretization
  - Predictor step (momentum eqns.): low–storage Runge–Kutta scheme, $\mathcal{O}(\Delta t^2)$
  - Corrector step (pressure–correction equation): SIP solver (ILU)

- Pressure–velocity coupling: Momentum interpolation of Rhie & Chow (1983)

- Various subgrid-scale and wall models

- High–performance computing techniques
  - Vectorized and parallelized

Assumptions

- Lagrangian frame of reference for the dispersed phase
- High volume fraction possible $\Rightarrow$ two- and four-way coupling
- Low density ratio of the bubbles: $\rho_{\mathrm{b}}/\rho_{\mathrm{f}} \ll 1$

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Newton's second law:

$$\left.\begin{aligned} \frac{\mathrm{d}\mathbf{u}_\mathrm{b}}{\mathrm{d}t} &= \frac{1}{m_\mathrm{b}} \sum_i \mathbf{F_i} \\ \frac{\mathrm{d}\mathbf{x}_\mathrm{b}}{\mathrm{d}t} &= \mathbf{u}_\mathrm{b} \end{aligned}\right\}$$

$$\frac{\mathrm{d}\boldsymbol{\omega}_\mathrm{b}}{\mathrm{d}t} = \frac{1}{I_\mathrm{b}} \mathbf{T}$$

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General procedure:

Assumptions

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General procedure:

Interpolation of $\mathbf{u}_\mathrm{f}$ to bubble location

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- Lagrangian frame of reference for the dispersed phase
- High volume fraction possible $\Rightarrow$ two- and four-way coupling
- Low density ratio of the bubbles: $\rho_\mathrm{b}/\rho_\mathrm{f} \ll 1$

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$$\frac{\mathrm{d}\boldsymbol{\omega}_\mathrm{b}}{\mathrm{d}t} = \frac{1}{I_\mathrm{b}}\mathbf{T}$$

General procedure:

Interpolation of $\mathbf{u}_\mathrm{f}$ to bubble location

$\longleftarrow$ 4th order Runge-Kutta scheme

$\longleftarrow$ Analytic solution

All[1] forces needed!

- Drag

$$\mathbf{F}_{\mathrm{D}} = \frac{1}{8} C_{\mathrm{D}} \rho_{\mathrm{f}} \pi d_{\mathrm{b}}^2 \left| \mathbf{u}_{\mathrm{f}} - \mathbf{u}_{\mathrm{b}} \right| \left(\mathbf{u}_{\mathrm{f}} - \mathbf{u}_{\mathrm{b}} \right)$$

  - Spherical bubble:

$$C_{\mathrm{D,s}} = \begin{cases} \frac{16}{\mathrm{Re_b}} \left[1 + 2/\left(1 + \frac{16}{\mathrm{Re_b}} + \frac{3.315}{\sqrt{\mathrm{Re_b}}} \right) \right] & \text{clean} \\ \frac{24}{\mathrm{Re_b}} \left[1 + 0.15 \mathrm{Re_b}^{0.687} \right] & \text{contam.} \end{cases}$$

  - Ellipsoidal bubble:

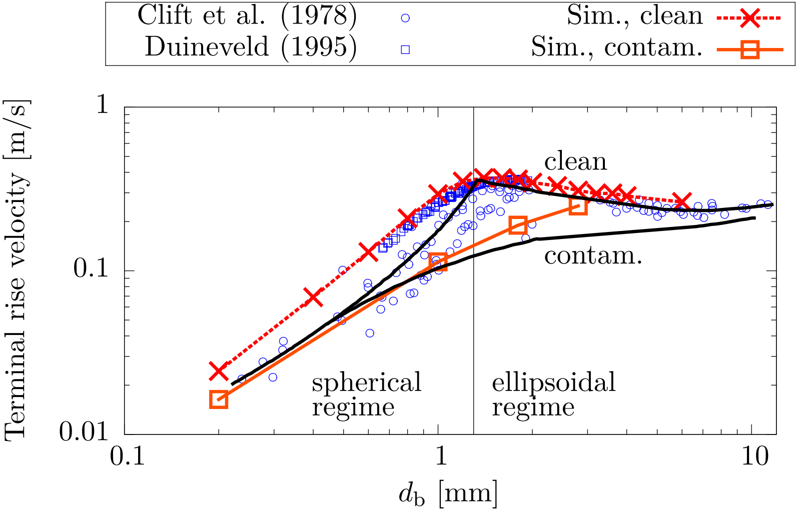
$$C_{\mathrm{D,e}} = 4\mathrm{Eo} / \left(\mathrm{Eo} + 9.5 \right)$$

$\Rightarrow$ Combine:

$$C_{\mathrm{D}} = \sqrt{C_{\mathrm{D,s}}^2 + C_{\mathrm{D,e}}^2}$$

[1] Except the Basset history force

Test: single rising bubble in a resting fluid



All[1] forces needed!

- Buoyancy + Gravity

$$\boxed{\mathbf{F}_{\mathrm{B}} + \mathbf{F}_{\mathrm{G}} = \mathbf{g}\,V_{\mathrm{b}}\,(\rho_{\mathrm{b}} - \rho_{\mathrm{f}})}$$

- Pressure gradient

$$\boxed{\mathbf{F}_{\mathrm{PG}} = \rho_{\mathrm{f}}\,V_{\mathrm{b}}\frac{\mathrm{D}\mathbf{u}_{\mathrm{f}}}{\mathrm{D}t}}$$

- Added-mass

$$\boxed{\mathbf{F}_{\mathrm{AM}} = \frac{1}{2}\rho_{\mathrm{f}}\,V_{\mathrm{b}}\left(\frac{\mathrm{D}\mathbf{u}_{\mathrm{f}}}{\mathrm{D}t} - \frac{\mathrm{D}\mathbf{u}_{\mathrm{b}}}{\mathrm{D}t}\right)}$$

[1] Except the Basset history force

All[1] forces needed!

- Lift

$$\boxed{\mathbf{F}_\mathrm{L} = C_\mathrm{L} \rho_\mathrm{b} V_\mathrm{b} \left(\mathbf{u}_\mathrm{f} - \mathbf{u}_\mathrm{b}\right) \times \mathrm{rot}\, \mathbf{u}_\mathrm{f}}$$

  - $\mathrm{Re_b} \ll 1$:

$$C_\mathrm{L,s} = 6/\pi^2 \left(\mathrm{Re_b Sr}\right)^{-\frac{1}{2}} \; \frac{2.255}{\left(1 + 0.2\mathrm{Re_b/Sr}\right)^{\frac{3}{2}}}$$

  - $\mathrm{Re_b} \gg 1$:

$$C_\mathrm{L,l} = 1/2 \frac{1 + 16/\mathrm{Re_b}}{1 + 29/\mathrm{Re_b}}$$

$\Rightarrow$ Combine:

$$C_\mathrm{D} = \sqrt{C_\mathrm{L,s}^2 + C_\mathrm{L,l}^2}$$

[1] Except the Basset history force

Challenge for LES:

- Fluid solver provides filtered fluid velocity $\overline{\mathbf{u}}_\mathrm{f}$
- Full velocity needed: $\mathbf{u}_\mathrm{f} = \overline{\mathbf{u}}_\mathrm{f} + \mathbf{u}'_\mathrm{f}$

Challenge for LES:

- Fluid solver provides filtered fluid velocity $\overline{\mathbf{u}}_\mathrm{f}$
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↓

Subgrid-scale model of Pozorski and Apte (2009)

$$d\mathbf{u}'_\mathrm{s} = -\mathbf{G}\,\mathbf{u}'_\mathrm{s}\,dt + \sqrt{2\sigma^2_\mathrm{SGS}}\,\mathbf{B}\,d\mathbf{W}$$

Langevin eq. $\rightarrow$ $\mathbf{u}'_\mathrm{f}$ at bubble position

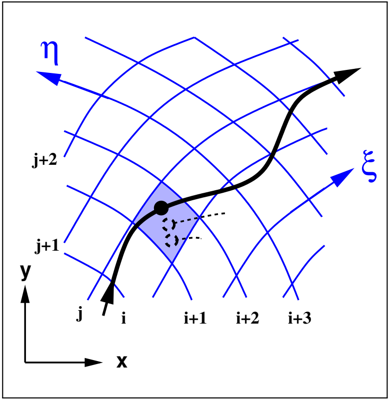
- Randomness $\longleftarrow$ Stochastic diff. term
- Temporal coupling $\longleftarrow$ Drift term
  - Crossing trajectory effect
  - Continuity effect
- Directional coupling $\longleftarrow$ Matrix form

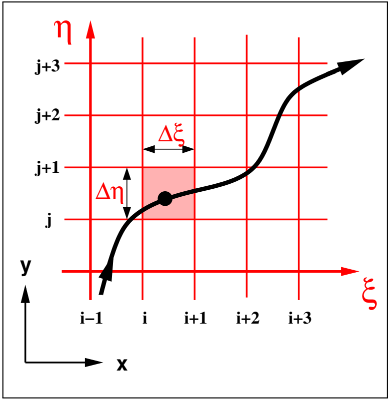
Subgrid-scale model of Pozorski and Apte (2009)

$$d\mathbf{u}'_{\mathrm{s}} = -\mathbf{G}\,\mathbf{u}'_{\mathrm{s}}\,dt + \sqrt{2\sigma^2_{\mathrm{SGS}}}\,\mathbf{B}\,d\mathbf{W}$$

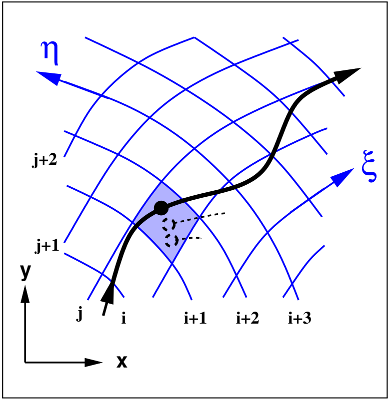
- Turbulent kinetic energy: $k_{\mathrm{SGS}} = 1/2\left(\overline{\mathbf{u}}_{\mathrm{f}} - \overline{\overline{\mathbf{u}}}_{\mathrm{f}}\right)^2$
- Estimated fluctuations: $\sigma_{\mathrm{SGS}} = \sqrt{2/3\,k_{\mathrm{SGS}}}$
- Fluctuation length scale: Δ_{SGS}
  - Δ_{filter} (Pozorski and Apte, 2009)
  - **Here: $\boldsymbol{\Delta}_{\mathbf{filter}}\,\mathbf{f}_{\mathbf{van\ Driest}}$**
- Fluctuation time scale: $\tau'_{\mathrm{L}} = C\,\frac{\Delta_{\mathrm{SGS}}}{\sigma_{\mathrm{SGS}}}$
- Crossing trajectory & continuity effect $\leftarrow$ Csanady (1963)
- Matrix $\mathbf{G} \Rightarrow \mathbf{B} = \sqrt{\mathbf{G}}$

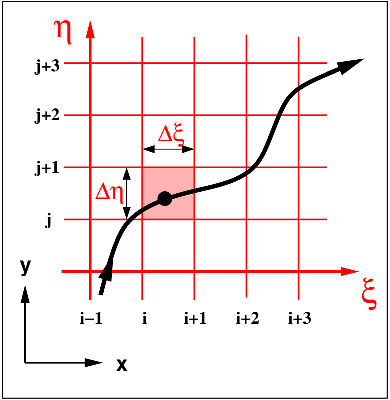
$$G_{ij} = \frac{1}{\tau'_{\mathrm{L},\perp}}\delta_{ij} + \left(\frac{1}{\tau'_{\mathrm{L},\parallel}} - \frac{1}{\tau'_{\mathrm{L},\perp}}\right) r_i\, r_j, \qquad r_i = \frac{u_{\mathrm{rel},i}}{|\mathbf{u}_{\mathrm{rel}}|}$$





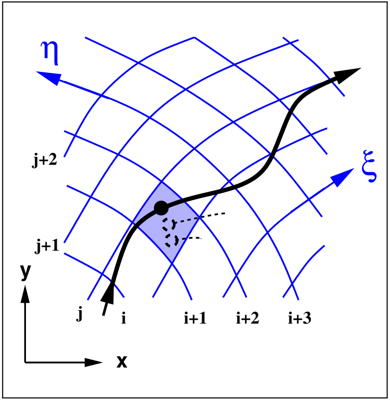
C-Space vs. P-Space Tracking



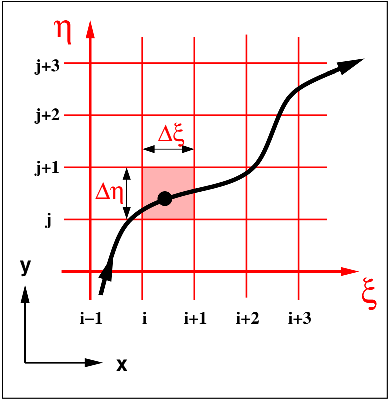


- P-space is curvilinear
- Point location not trivial

⇒ Time-consuming search algorithms required



Trans
⟺
form



- P-space is curvilinear
- Point location not trivial

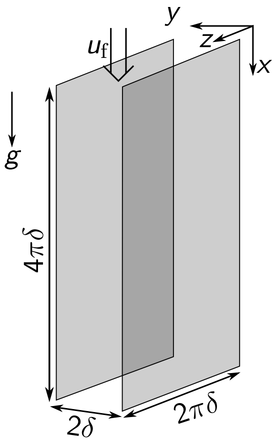
⇒ Time-consuming search algorithms required

- C-space is orthonormal
- Point location trivial

⇒ No search algorithms required

C-Space vs. P-Space Tracking

DNS:
Channel downflow of Molin et al. (2012)



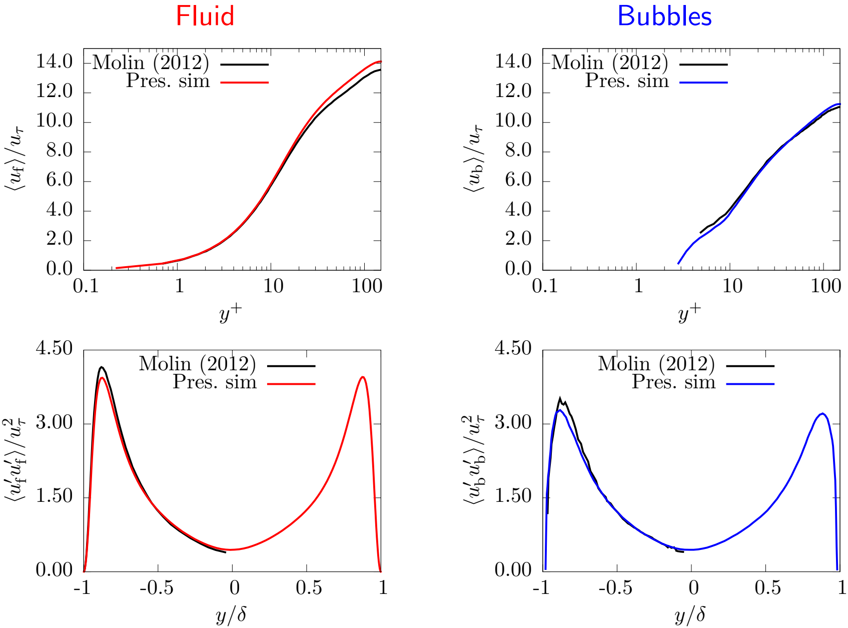
Present Simulation:

Flow

- $\mathrm{Re}_\tau = u_\tau\, \delta/\nu = 150$, $\delta = 20\,\mathrm{mm}$
- Fixed pressure gradient
- Grid: $256 \times 128 \times 256$ CV
- Periodic BC stream- and spanwise
- No-slip BC at smooth wall
- Two-way coupled

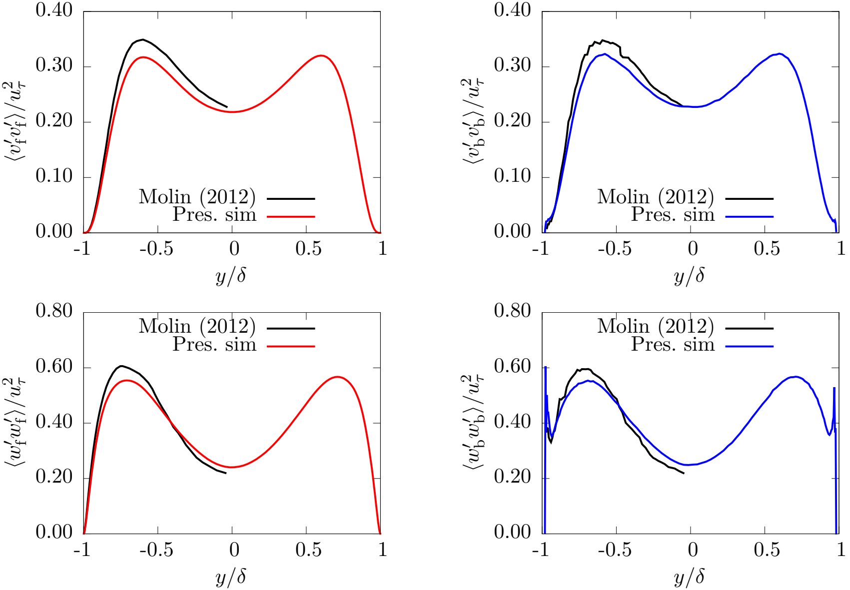
Bubbles

- $N_{\mathrm{b}} = 21,940$
- $d_{\mathrm{b}} = 220\,\mu\mathrm{m}$
- $\Phi_{\mathrm{V,tot}} = 1 \cdot 10^{-4}$
- Surfactant contaminated
- Bubble SGS model not activated



Comparison with the Reference Case

Fluid | Bubbles



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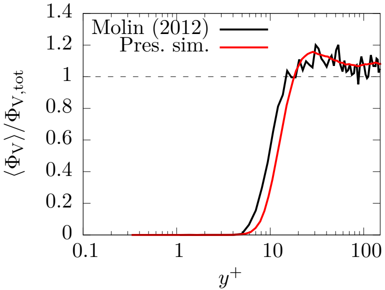
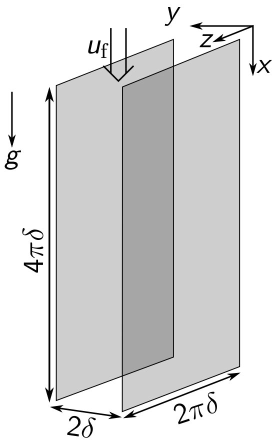


- Slight overestimation of the velocities in the channel center
- Fluctuations slightly underpredicted

- Bubbles follow fluid motion
- Close agreement for volume fraction

⇒ Close overall agreement between present sim. and reference data

DNS:
Channel downflow of
Molin et al. (2012)



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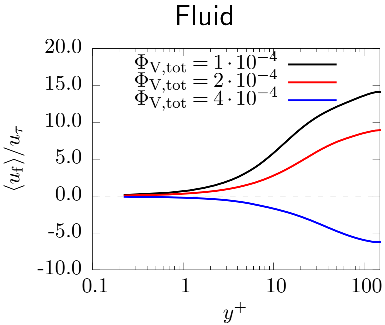
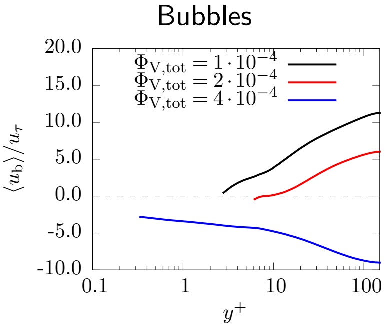
- Two-way coupled

Bubbles

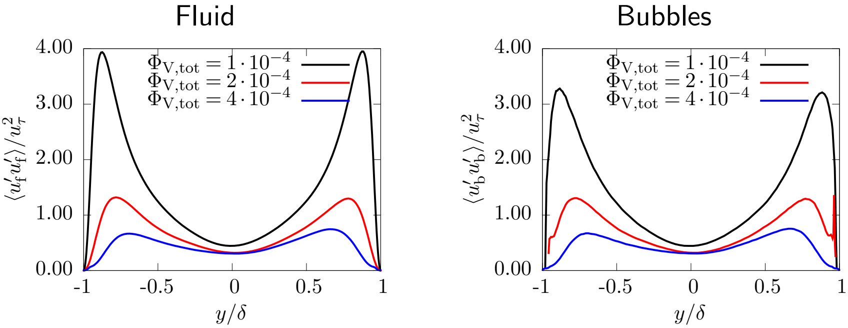
- $N_\mathrm{b} = 21,940/43,880/87,760$
- $d_\mathrm{b} = 220\,\mu\mathrm{m}$
- $\Phi_{\mathrm{V,tot}} = 1 \cdot 10^{-4}/2 \cdot 10^{-4}/4 \cdot 10^{-4}$
- Surfactant contaminated
- Bubble SGS model not activated





- $\Phi_{V,tot} = 2 \times 10^{-4}$: strongly reduced fluid velocity
- $\Phi_{V,tot} = 4 \times 10^{-4}$: reversed flow $\rightarrow$ upflow

$\Rightarrow$ Strong impact of momentum added by two-way coupling

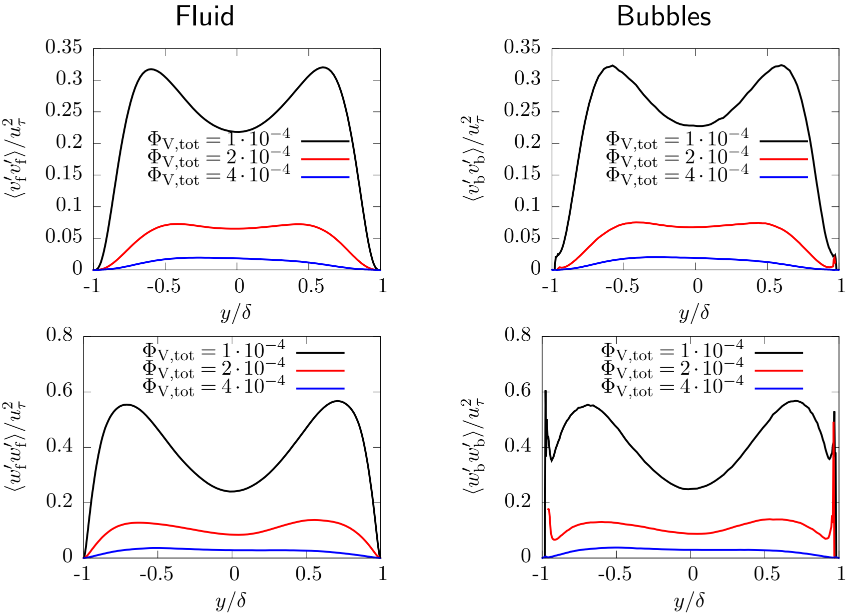


Higher volume fraction

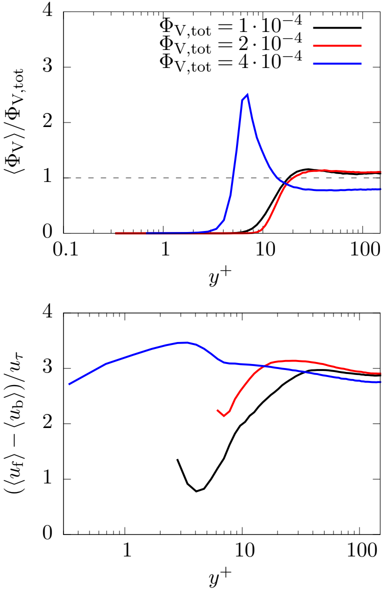
- Fluctuations strongly reduced
- Bubble and fluid fluctuations equal

  $\rightarrow$ Fluid fluctuations driven by the bubbles

$\Rightarrow$ Laminarization



Increased Volume Fraction



- $\Phi_{V,tot} = 2 \times 10^{-4}$: bubbles pushed towards channel center

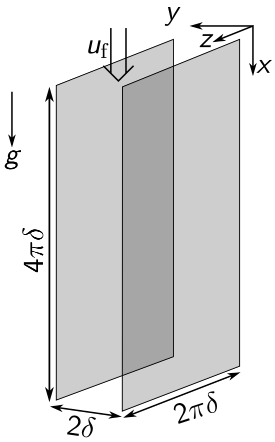
$\rightarrow$ Stronger lift due to higher u_{rel}

- $\Phi_{V,tot} = 4 \times 10^{-4}$:
  - Peak at $y^+ \approx 7$

Video: $\Phi_{tot} = 1 \cdot 10^{-4}$

Video: $\Phi_{tot} = 4 \cdot 10^{-4}$

DNS:
Channel downflow of
Molin et al. (2012)



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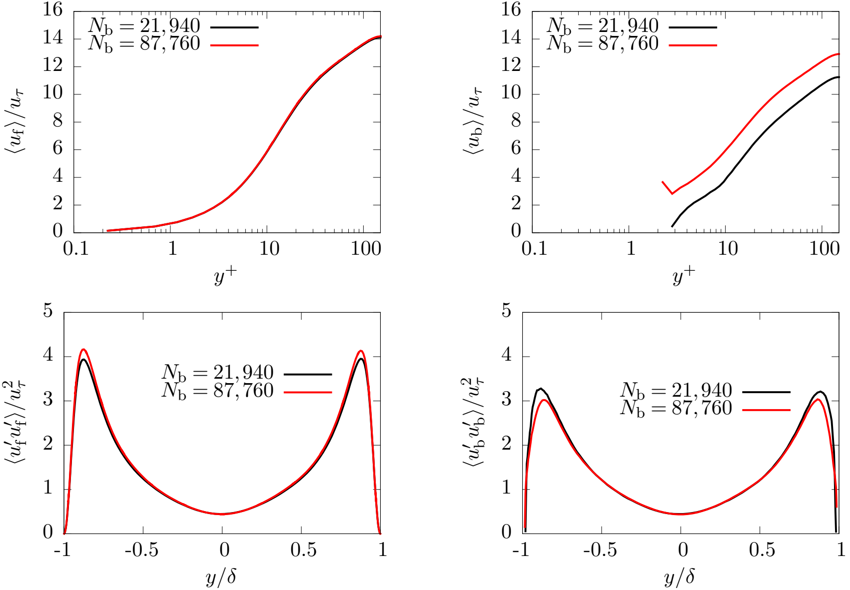
- No-slip BC at smooth wall
- Two-way coupled

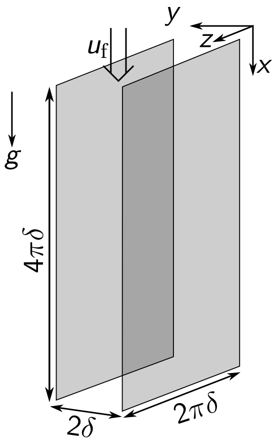
Bubbles

- $N_\mathrm{b} = 21,940/87,760$
- $d_\mathrm{b} = 220\,\mu\mathrm{m}/d_\mathrm{b} = 138.6\,\mu\mathrm{m}$
- $\Phi_{\mathrm{V,tot}} = 1 \cdot 10^{-4}$
- Surfactant contaminated
- Bubble SGS model not activated



Increased Bubble Number

DNS:
Channel downflow of
Molin et al. (2012)



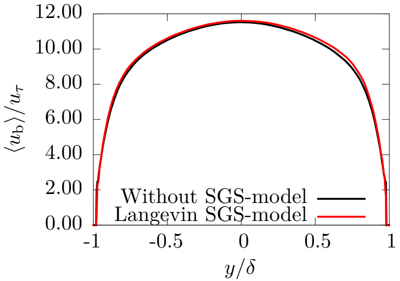
Present Simulation:

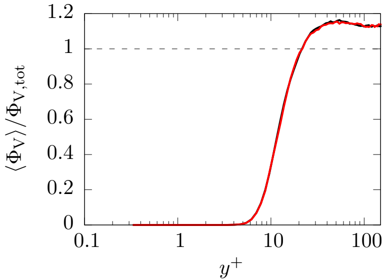
Flow

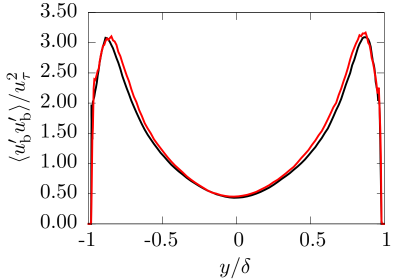
- $\mathrm{Re}_\tau = u_\tau\,\delta/\nu = 150$, $\delta = 20\,\mathrm{mm}$
- Fixed pressure gradient
- Grid: 128 × 128 × 128 CV
- Periodic BC stream- and spanwise
- No-slip BC at smooth wall
- Two-way coupled

Bubbles

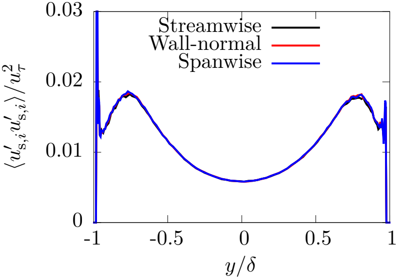
- $N_\mathrm{b} = 21,940$
- $d_\mathrm{b} = 220\,\mu\mathrm{m}$
- $\Phi_{\mathrm{V,total}} = 1 \cdot 10^{-4}$
- Surfactant contaminated
- Particle SGS model of Pozorski & Apte (2009) activated

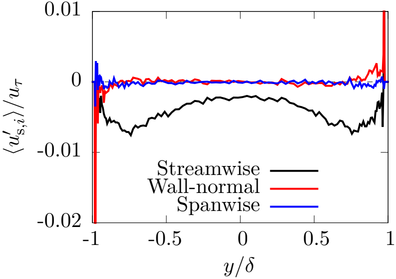


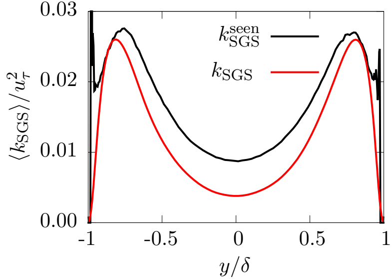


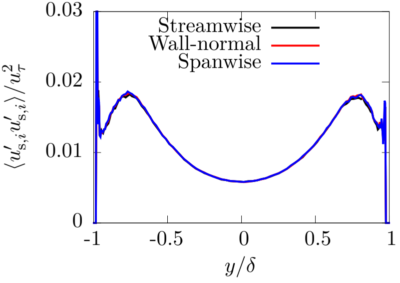


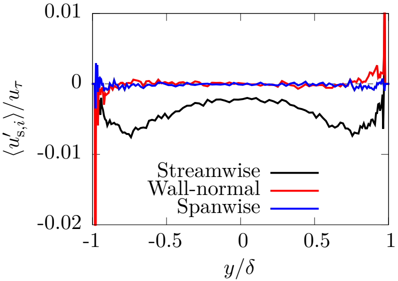
- Similar results for wall-normal and spanwise directions

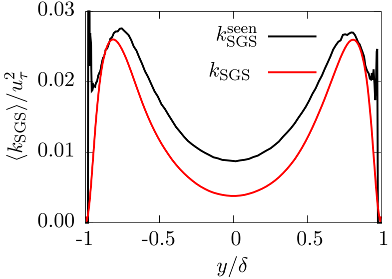

Streamwise
Wall-normal
Spanwise
0.03
0.02
0.01
0
-1
-0.5
0
0.5
1
$\langle u'_{s,i} u'_{s,i} \rangle / u_\tau^2$
y/δ


0.01
0
-0.01
-0.02
-1
-0.5
0
0.5
1
Streamwise
Wall-normal
Spanwise
$\langle u'_{s,i} \rangle / u_\tau$
y/δ

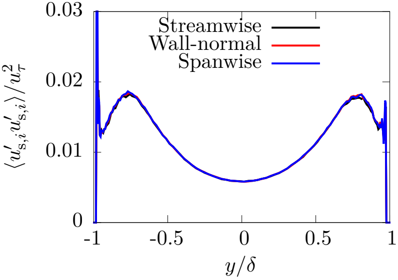

$k_{\mathrm{SGS}}^{\mathrm{seen}}$
k_{SGS}
0.03
0.02
0.01
0.00
-1
-0.5
0
0.5
1
$\langle k_{\mathrm{SGS}} \rangle / u_\tau^2$
y/δ

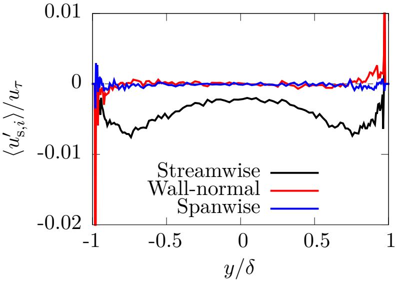


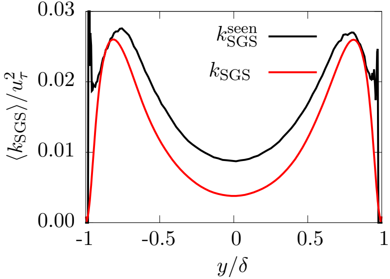




- Correct level of turbulent kinetic energy added







- Correct level of turbulent kinetic energy added
- $k_{SGS}^{seen} > k_{SGS}$
- Bubbles tend to accumulate in regions of high fluctuations/turbulent kinetic energy

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Influence of the Particle Subgrid-Scale Model

Conclusions

- LESOCC successfully extended towards tracking of bubbles
  → Very good agreement to DNS results
- Increased volume fraction
  - Decreases/reverts fluid velocity
  - Laminarizes the flow
- Increased bubble number with marginal effect
- Subgrid-scale model for the dispersed phase
  → Small influence for bubbles

Outlook

- Extend present investigations
- Higher Re flows
- Include coalescence & break-up of bubbles

Thank you for your attention!

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Backup

Subgrid-scale model of Pozorski and Apte (2009)

$$d\mathbf{u}'_{\mathrm{s}} = -\mathbf{G}\,\mathbf{u}'_{\mathrm{s}}\,dt + \sqrt{2\sigma^2_{\mathrm{SGS}}}\,\mathbf{B}\,d\mathbf{W}$$

- Turbulent kinetic energy: $k_{\mathrm{SGS}} = 1/2\left(\overline{\mathbf{u}}_{\mathrm{f}} - \overline{\overline{\mathbf{u}}}_{\mathrm{f}}\right)^2$
- Estimated fluctuations: $\sigma_{\mathrm{SGS}} = \sqrt{2/3\,k_{\mathrm{SGS}}}$
- Fluctuation length scale: Δ_{SGS}
  - Δ_{filter} (Pozorski and Apte, 2009)
  - **Here: $\Delta_{\mathrm{filter}}\, f_{\mathrm{van\ Driest}}$**
- Fluctuation time scale: $\tau'_{\mathrm{L}} = C\,\frac{\Delta_{\mathrm{SGS}}}{\sigma_{\mathrm{SGS}}}$

Subgrid-scale model of Pozorski and Apte (2009)

$$d\mathbf{u}'_{\mathrm{s}} = -\mathbf{G}\,\mathbf{u}'_{\mathrm{s}}\,dt + \sqrt{2\sigma^2_{\mathrm{SGS}}}\,\mathbf{B}\,d\mathbf{W}$$

- Crossing trajectory & continuity effect $\leftarrow$ Csanady (1963)
  - Parallel to direction of relative motion

$$\tau'_{\mathrm{L},\parallel} = \frac{\tau'_{\mathrm{L}}}{\sqrt{1 + \mathbf{u}^2_{\mathrm{rel}}/\sigma^2_{\mathrm{rel}}}}$$

  - Perpendicular to direction of relative motion

$$\tau'_{\mathrm{L},\perp} = \frac{\tau'_{\mathrm{L}}}{\sqrt{1 + 4\mathbf{u}^2_{\mathrm{rel}}/\sigma^2_{\mathrm{rel}}}}$$

Subgrid-scale model of Pozorski and Apte (2009)

$$d\mathbf{u}'_{\mathrm{s}} = -\mathbf{G}\,\mathbf{u}'_{\mathrm{s}}\,dt + \sqrt{2\sigma^2_{\mathrm{SGS}}}\,\mathbf{B}\,d\mathbf{W}$$

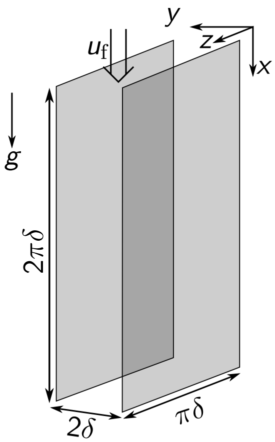
- Matrix $\mathbf{G}$

$$G_{ij} = \frac{1}{\tau'_{\mathrm{L},\perp}}\delta_{ij} + \left(\frac{1}{\tau'_{\mathrm{L},\parallel}} - \frac{1}{\tau'_{\mathrm{L},\perp}}\right) r_i\, r_j$$

- Matrix $\mathbf{B} = \sqrt{\mathbf{G}}$

$$B_{ij} = \frac{1}{\sqrt{\tau'_{\mathrm{L},\perp}}}\delta_{ij} + \left(\frac{1}{\sqrt{\tau'_{\mathrm{L},\parallel}}} - \frac{1}{\sqrt{\tau'_{\mathrm{L},\perp}}}\right) r_i\, r_j$$

- Direction of relative motion $r_i = u_{\mathrm{rel},i}/|\mathbf{u}_{\mathrm{rel}}|$



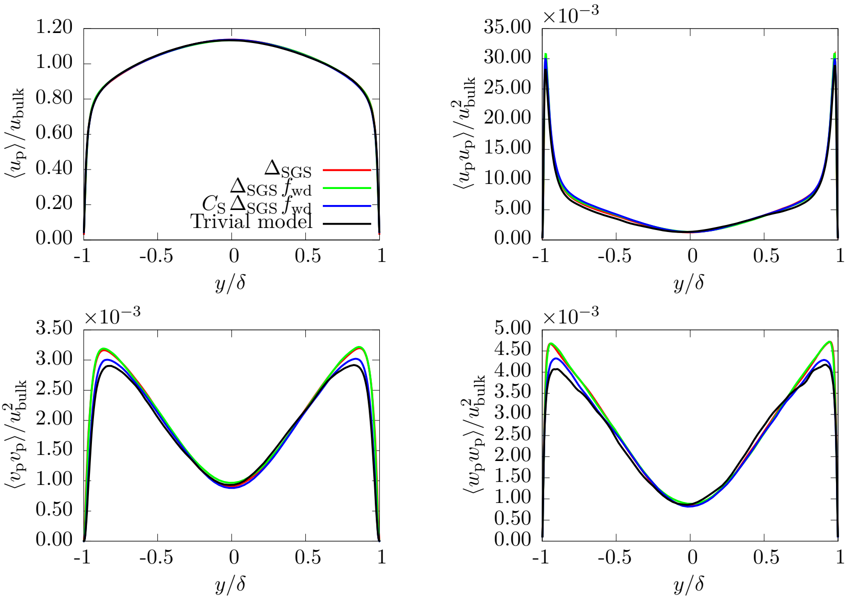
Present Simulation:

Flow

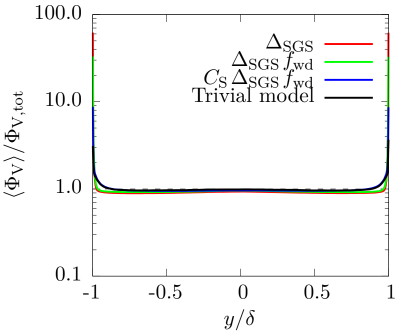
- $\mathrm{Re}_\tau = u_\tau \delta/\nu = 644$, $\delta = 20\,\mathrm{mm}$
- Grid: $128 \times 128 \times 128$ CV
- Periodic BC stream- and spanwise
- No-slip BC at smooth wall
- Four-way coupled

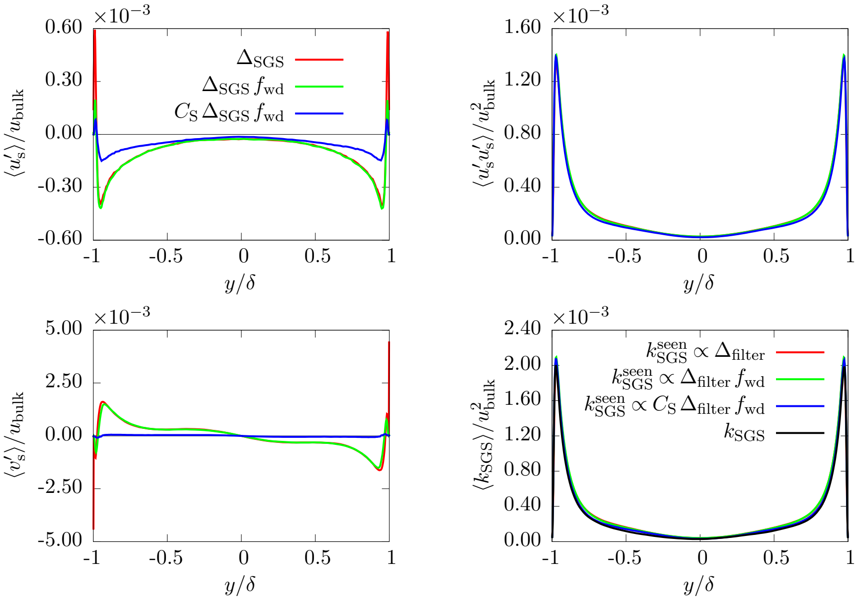
Particles

- $N_\mathrm{p} = 6 \cdot 10^6$
- $d_\mathrm{b} = 4\,\mathrm{\mu m}$
- $\Phi_\mathrm{V,tot} = 6.78 \cdot 10^{-7}$
- $\eta = 1.23 \cdot 10^{-3}$
- SGS model of Pozorski & Apte (2009)



$\langle u_p \rangle / u_{bulk}$
y/δ
Δ_{SGS}
$\Delta_{SGS} f_{wd}$
$C_S \Delta_{SGS} f_{wd}$
Trivial model
$\times 10^{-3}$
$\langle u_p u_p \rangle / u_{bulk}^2$
$\langle v_p v_p \rangle / u_{bulk}^2$
$\langle w_p w_p \rangle / u_{bulk}^2$


100.0
10.0
1.0
0.1
-1
-0.5
0
0.5
1
$\langle \Phi_V \rangle / \Phi_{V,tot}$
y/δ
Δ_{SGS}
$\Delta_{SGS} f_{wd}$
$C_S \Delta_{SGS} f_{wd}$
Trivial model



$\times 10^{-3}$
0.60
0.30
0.00
-0.30
-0.60
$\langle u'_\mathrm{s} \rangle / u_\mathrm{bulk}$
Δ_SGS
$\Delta_\mathrm{SGS}\, f_\mathrm{wd}$
$C_\mathrm{S}\, \Delta_\mathrm{SGS}\, f_\mathrm{wd}$
-1
-0.5
0
0.5
1
y/δ
$\times 10^{-3}$
1.60
1.20
0.80
0.40
0.00
$\langle u'_\mathrm{s} u'_\mathrm{s} \rangle / u^2_\mathrm{bulk}$
-1
-0.5
0
0.5
1
y/δ
$\times 10^{-3}$
5.00
2.50
0.00
-2.50
-5.00
$\langle v'_\mathrm{s} \rangle / u_\mathrm{bulk}$
-1
-0.5
0
0.5
1
y/δ
$\times 10^{-3}$
2.40
2.00
1.60
1.20
0.80
0.40
0.00
$\langle k_\mathrm{SGS} \rangle / u^2_\mathrm{bulk}$
$k^\mathrm{seen}_\mathrm{SGS} \propto \Delta_\mathrm{filter}$
$k^\mathrm{seen}_\mathrm{SGS} \propto \Delta_\mathrm{filter}\, f_\mathrm{wd}$
$k^\mathrm{seen}_\mathrm{SGS} \propto C_\mathrm{S}\, \Delta_\mathrm{filter}\, f_\mathrm{wd}$
k_SGS
-1
-0.5
0
0.5
1
y/δ