Multiphase Flow of Natural Gas through Pipelines

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Agenda

- Motivation
- Physical problem
- Mathematical model
  - Single-phase gas flow
  - Two-phase particle laden gas flow
- Numerical approximation
  - Stabilized Finite element method
  - Shock tube problem
- Future work

Motivation

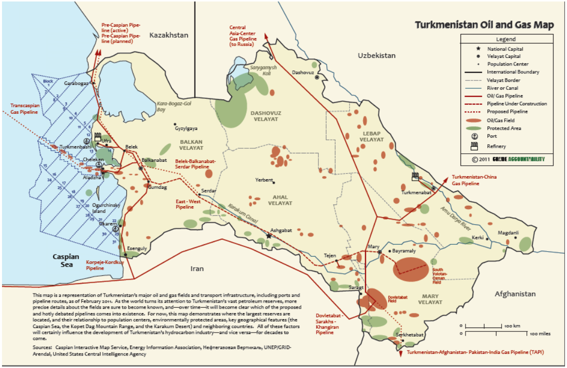


Figure: Oil and Gas fields and pipelines of Turkmenistan

Motivation

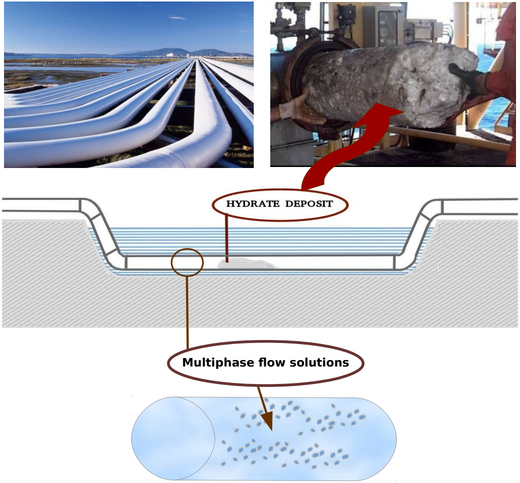


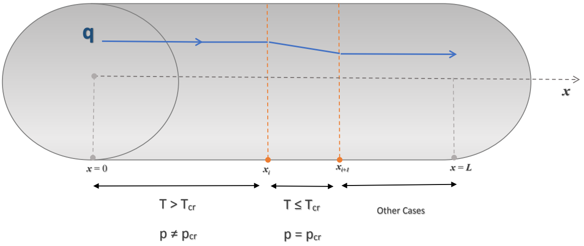
Figure: Description of the Physical problem

Physical problem

- Gas transportation requires interconnected systems of pipelines with hundreds or thousands of kilometres of pipes.
- Such pipelines can transport natural gas through it and can be located under the water surface.
- During transportation, there is a heat transfer between gas and the environment.
- Some parts of gas components can undergo a phase change such that gas hydrates are formed.
- This leads to an unbalance in the flow rate of the gas loss of 1-3 % from the total volume of transfered gas.
- Multiphase flow has to be studied.

Flow rate in a horizontal pipeline

- For $T > T_{cr}$ and $p \neq p_{cr}$ pure gas flow,
- For $T \leqslant T_{cr}$ and $p = p_{cr}$ formation of gas hydrates,
- For other cases gas flow with hydrates.



Mathematical model: Single-phase gas flow

Euler equations of compressible flow:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \sum_{j=1}^{2} \frac{\partial \boldsymbol{F}_j(\boldsymbol{U})}{\partial x_j} = 0 \qquad \text{in} \quad Q_l = \Omega \times (0, l)$$

where $\Omega \in \Re^2$ is a bounded domain occupied be gas, $l > 0$ is the length of a time interval. With the state vector

$$\boldsymbol{U} = (u_1, u_2, u_3, u_4)^T = (\rho, \rho v_1, \rho v_2, E)^T$$

and Euler fluxes

$$\boldsymbol{F}(\boldsymbol{U}) = \begin{pmatrix} \rho v_1 & \rho v_2 \\ \rho v_1^2 + p & \rho v_1 v_2 \\ \rho v_1 v_2 & \rho v_2^2 + p \\ (E+p) v_1 & (E+p) v_2 \end{pmatrix}$$

Mathematical model: Single-phase gas flow

- Equation of state

$$p = (\gamma - 1)(E - \frac{\rho|\boldsymbol{v}|^2}{2})$$

with the Poisson adiabatic constant $\gamma > 1$.

- Initial conditions

$$\boldsymbol{U}(x, 0) = \boldsymbol{U}^0(x), \quad x \in \Omega$$

- Boundary conditions (inflow, outflow, fixed wall condition, moving walls, periodic, farfield)

$$\boldsymbol{B}(\boldsymbol{U}) = G, \quad \text{on} \quad \partial\Omega \times (0, l),$$

where B is some boundary operator.

Mathematical model: Single-phase gas flow

Assuming that $\boldsymbol{U} \in C^1(Q_T)^m$, system of conservation laws can be written as a quasilinear system

$$\frac{\partial \boldsymbol{U}}{\partial t} + \sum_{j=1}^{m} \boldsymbol{A}_j(\boldsymbol{U}) \frac{\partial \boldsymbol{U}}{\partial x_j} = 0$$

with $m \times m$ matrices $\boldsymbol{A}_j(\boldsymbol{U}), j = 0, ..., m$, which depend on the unknown function $\boldsymbol{U}$ in a generally nonlinear way. Here,

$$\boldsymbol{A}_j = \frac{\partial \boldsymbol{F}_j(\boldsymbol{U})}{\partial \boldsymbol{U}}$$

is the Jacobi matrix.

Remark. *Nonlinear phenomena as nonexistence of global smooth solutions on a massive set of initial and boundary data may occur.*

Mathematical model: Two-phase flow

Two-fluid flow model: two flow phases of gas and dispersed particles.

Gas phase:

$$\frac{\partial(\alpha_g \rho_g)}{\partial t} + \nabla\cdot(\alpha_g \rho_g \boldsymbol{v}_g) = 0$$

$$\frac{\partial(\alpha_g \rho_g \boldsymbol{v}_g)}{\partial t} + \nabla\cdot(\alpha_g \rho_g \boldsymbol{v}_g \otimes \boldsymbol{v}_g) + \nabla(\alpha_g p_g) = p_g^i \nabla \alpha_g - \boldsymbol{f}_d$$

$$\frac{\partial(\alpha_g E_g)}{\partial t} + \nabla\cdot((\alpha_g E_g + \alpha_g p_g)\boldsymbol{v}_g) = \boldsymbol{v}^i \cdot (p_g^i \nabla \alpha_g - \boldsymbol{f}_d) - q_h.$$

Solid phase (dispersed particles):

$$\frac{\partial(\alpha_s \rho_s)}{\partial t} + \nabla\cdot(\alpha_s \rho_s \boldsymbol{v}_s) = 0$$

$$\frac{\partial(\alpha_s \rho_s \boldsymbol{v}_s)}{\partial t} + \nabla\cdot(\alpha_s \rho_s \boldsymbol{v}_s \otimes \boldsymbol{v}_s) + \nabla(\alpha_s p_s) = p_s^i \nabla \alpha_s + \boldsymbol{f}_d$$

$$\frac{\partial(\alpha_s E_s)}{\partial t} + \nabla\cdot((\alpha_s E_s + \alpha_s p_s)\boldsymbol{v}_s) = \boldsymbol{v}^i \cdot (p_s^i \nabla \alpha_s + \boldsymbol{f}_d) + q_h.$$

Mathematical model: Two-phase flow

- Saturation constraint: $\alpha_g + \alpha_s = 1$
- Equation of state: $p = (\gamma - 1)\rho_g e_g$, $e_g = c_{\upsilon g} T_g$ where, e_g is the internal energy, $c_{\upsilon g}$ is the specific heats at constant volume and T_g is temperature.
- Interfacial drag force and the heat transfer:

$$\boldsymbol{f}_d = \frac{3C_d\alpha_s\rho_g}{4d}|\boldsymbol{u}_g - \boldsymbol{u}_s|(\boldsymbol{u}_g - \boldsymbol{u}_s), \quad C_d = \begin{cases} \frac{24}{Re}\left(1 + 0.15Re^{0.687}\right), & Re < 1000, \\ 0.44, & Re \geq 1000. \end{cases}$$

$$q_h = \frac{6Nu}{d^2}k_g\alpha_s(T_g - T_s), \quad Nu = 2 + 0.65Re^{\frac{1}{2}}\left(\frac{c_{pg}\mu_g}{k_g}\right)^{\frac{1}{3}}.$$

Here, C_d, Re, Nu, k_g, c_{pg}, μ_g are a dimensionless drag coefficient, Reynolds number, Nusselt number, thermal conductivity, heat capacity at constant pressure and (microscopic) dynamic viscosity, respectively.

Mathematical model: Continuous problem

- **Vector form**

$$\frac{\partial \boldsymbol{U}}{\partial t} + \nabla \cdot \boldsymbol{F}(\boldsymbol{U}) = \boldsymbol{G}$$

where, $\boldsymbol{F}(\boldsymbol{U}) = (\boldsymbol{F}_1(\boldsymbol{U}), \boldsymbol{F}_2(\boldsymbol{U}))^T$ are inviscid fluxes and $\boldsymbol{G}$ is force term.

- **Quasilinear form**

(For simplicity $\boldsymbol{G} = 0$)

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{A}_j(\boldsymbol{U}) \nabla \boldsymbol{U} = \boldsymbol{0}$$

where,

$$\boldsymbol{A}_j(\boldsymbol{U}) = \frac{\partial \boldsymbol{F}_j(\boldsymbol{U})}{\partial \boldsymbol{U}}, \quad j = 1, 2.$$

Numerical approximation

- Two-fluid model, each phase is the single-phase hyperbolic conservation laws,
- A classical solution of nonlinear hyperbolic systems is rather complex,
- For singularly perturbed parabolic and hyperbolic equations the standard application of the finite element method gives nonphysical oscillations,
- Streamline upwind Petrov-Galerkin method, where the artificial diffusion acting only in the direction of the streamlines,
- Approximate solution by usung SUPG may still exhibit overshoots or undershoots,
- Shock capturing method, where the artificial diffusion acting perpendicular to the streamlines.

Numerical approximation

Finite element spaces

- $V_h \subset H^1(\Omega) = \{u \in L^2(\Omega) : \nabla u \in L^2(\Omega)\}$,
- $S_h = \{\boldsymbol{U}_h | \boldsymbol{U}_h \in [V_h]^m, \boldsymbol{U}_h|_e \in [P^1(e)]^m, e \in \tau_h, \boldsymbol{B}(\boldsymbol{U_h}) = G, \quad \text{on} \quad \Gamma\}$,
- $W_h = \{\varphi_h | \varphi_h \in [V_h]^m, \varphi_h|_e \in [P^1(e)]^m, e \in \tau_h, \boldsymbol{B}(\varphi_{\boldsymbol{h}}^-) = 0, \quad \text{on} \quad \Gamma\}$

Assumption

- $\partial\Omega_h = \Gamma = \Gamma_I \cup \Gamma_O \cup \Gamma_W$

Numerical approximation

- The stabilized full-discrete weak form.
  Find $\boldsymbol{U}_h^n \in S_h$ *with* $\boldsymbol{U}_h(0) = P_h \boldsymbol{U}^0$ *such that:*

$$\int_\Omega \frac{\boldsymbol{U}_h^{n+1} - \boldsymbol{U}_h^n}{k} \cdot \varphi_h - \int_\Omega \boldsymbol{F}(\overline{\boldsymbol{U}}_h) \cdot \nabla \varphi_h + \int_\Gamma \boldsymbol{N}(\overline{\boldsymbol{U}}_h^+, \overline{\boldsymbol{U}}_h^-, \boldsymbol{n}) \cdot \varphi_h^+$$
$$+SUPG(\overline{\boldsymbol{U}}_h, \varphi_h) + SHOCK(\overline{\boldsymbol{U}}_h, \varphi_h) = 0$$

  $\forall \varphi_h \in W_h$.
  Here,

$$\overline{\boldsymbol{U}}_h = \frac{\boldsymbol{U}_h^{n+1} + \boldsymbol{U}_h^n}{2}$$

- The nonlinear system of equations is solved by Newton iteration.

Numerical approximation

We approximate the boundary flux with a numerical flux N, as numerical flux we choose Lax-Friedrichs flux, such that:

$$\boldsymbol{N}(\overline{\boldsymbol{U}}_h^+, \overline{\boldsymbol{U}}_h^-, \boldsymbol{n}) = \left(\boldsymbol{F}(\overline{\boldsymbol{U}}_h^+) \cdot \boldsymbol{n} + \boldsymbol{F}(\overline{\boldsymbol{U}}_h^-) \cdot \boldsymbol{n} + \alpha(\overline{\boldsymbol{U}}_h^+ - \overline{\boldsymbol{U}}_h^-) \right) /2$$

where $\boldsymbol{n}$ is normal vector, α is fixed number or mesh depend value.
The streamline diffusion stabilization term, such that:

$$SUPG(\overline{\boldsymbol{U}}_h, \varphi_h) = \sum_{e \in \tau_h} \int_e \delta_e \cdot \left(\frac{\boldsymbol{U}_h^{n+1} - \boldsymbol{U}_h^n}{k} + \boldsymbol{A}(\overline{\boldsymbol{U}}_h) \cdot \nabla \overline{\boldsymbol{U}}_h \right) \cdot \boldsymbol{A}(\overline{\boldsymbol{U}}_h) \cdot \nabla \varphi_h$$

where δ_e is SUPG stabilization parameter. The shock capturing term, such that:

$$SHOCK(\overline{\boldsymbol{U}}_h, \varphi_h) = \sum_{e \in \tau_h} \int_e \eta_e \cdot \nabla \overline{\boldsymbol{U}}_h \cdot \nabla \varphi_h$$

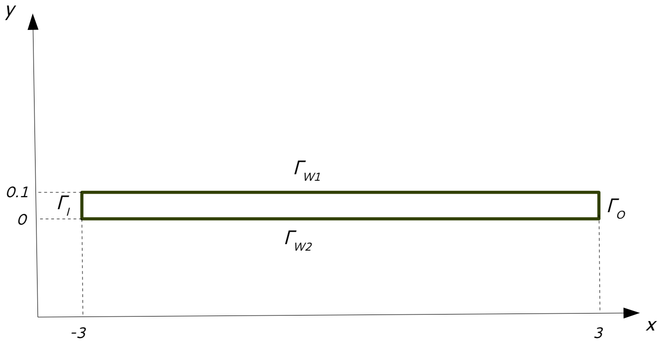
where η_e is shock-capturing parameter.

Shock tube problem: Single phase gas flow

The initial condition is given as:

$$\boldsymbol{U}^0 = (\rho^0, \rho^0 v_1^0, \rho^0 v_2^0, E^0)^T = \begin{cases} (1.0, 0.0, 0.0, 5.0)^T, & x < 0, \\ (1.101463, 0.0, 0.0, 2.5)^T, & x \geq 0. \end{cases}$$

The inflow/outflow and solid wall boundary conditions are applied on $\partial\Omega_h = \Gamma_I \cup \Gamma_O \cup \Gamma_{W1} \cup \Gamma_{W2}$



Shock tube problem: Single phase gas flow

At time $t = 1$ we compare the density distribution in the cut $y = 0$ computed by the stabilized FEM with analytical solution.

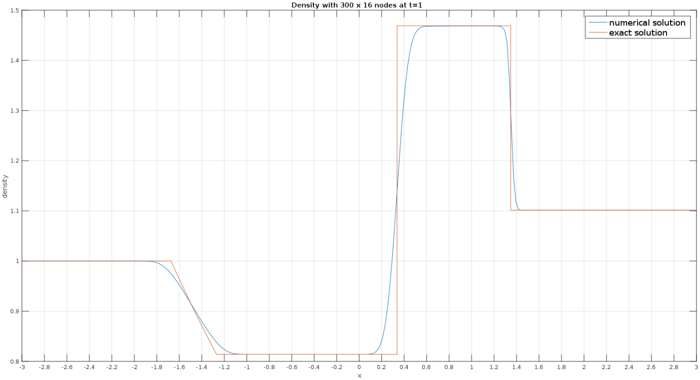


Figure: Density distribution at time $t = 1$ with 300 x 16 nodes.

Shock tube problem: Single phase gas flow

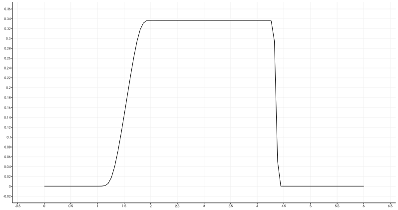


Figure: Velocity distribution at time $t = 1$ with 300 x 16 nodes.

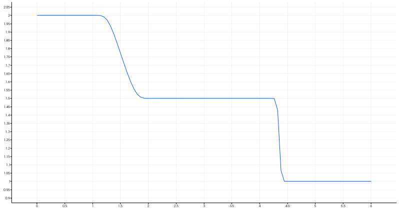


Figure: Pressure distribution at time $t = 1$ with 300 x 16 nodes.

Shock tube problem: two-phase flow

The initial dates:

$$\alpha_g^0 = \begin{cases} 0.25, & x < 0, \\ 0.10, & x \geq 0. \end{cases}$$

$$\boldsymbol{U}^0 = (\alpha_g^0 \rho_g^0, \alpha_g^0 \rho_g^0 v_{1g}^0, \alpha_g^0 \rho^0 v_{2g}^0, \alpha_g^0 E_g^0, \alpha_l^0, \alpha_l^0 v_{1l}^0, \alpha_l^0 v_{2l}^0, \alpha_l^0 E_l^0)^T =$$

$$\begin{cases} (0.25, 0.0, 0.0, 1.25, 0.75, 0.0, 0.0, 3.75)^T, & x < 0, \\ (0.1101463, 0.0, 0.0, 0.25, 0.9, 0.0, 0.0, 2.25)^T, & x \geq 0. \end{cases}$$

Shock tube problem: two-phase flow

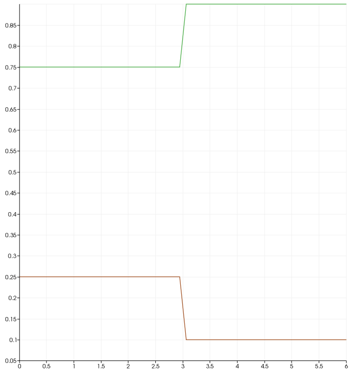


Figure: Initial volume fractions for gas (red) and for liquid (green).

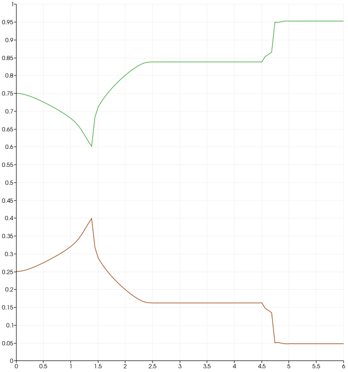


Figure: Volume fractions for gas (red) and for liquid (green) at $t = 1$.

Shock tube problem: two-phase flow

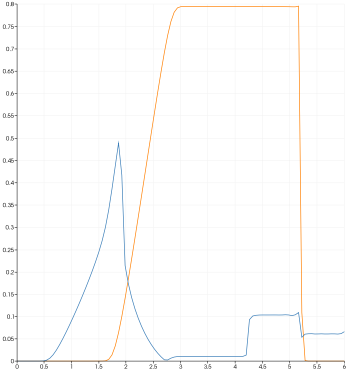


Figure: Velocity distributions for gas (orange) and liquid (blue) at time $t = 1$.

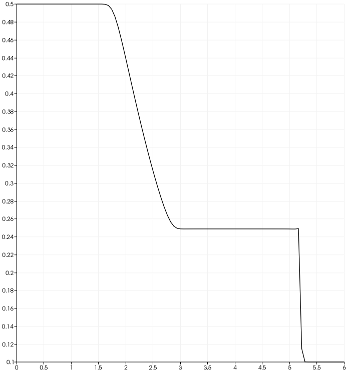
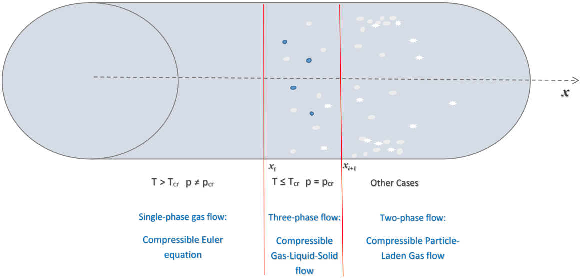


Figure: Pressure distributions at time $t = 1$.

Future work

- Städtke H., Gasdynamic Aspects of Two-Phase Flow, Wiley-VCH; 1 edition (October 6, 2006),
- Implementation and testing of the overall model,
- Improvement of stabilization techniques,
- Parallel implementation in three dimension,
- Three phase flow.



Thank you for your attention!