14th Workshop on Two-Phase Flow Predictions

Halle, 7-10 September 2015

Numerical Simulation of Elongated Fibres in Horizontal Channel Flow

Brian Quintero[a,b], Santiago Laín[a,b] and Martin Sommerfeld[a,b]

[a] Martin-Luther-Universität Halle-Wittenberg, Zentrum für Ingenieurwissenschaften, Mechanische Verfahrenstechnik, 06099 Halle (Saale), Germany.
www-mvt.iw.uni-halle.de

[b] Universidad Autonoma de Occidente (UAO), Energetics and Mechanics Department, FMRG, Cali, Colombia.

Contents

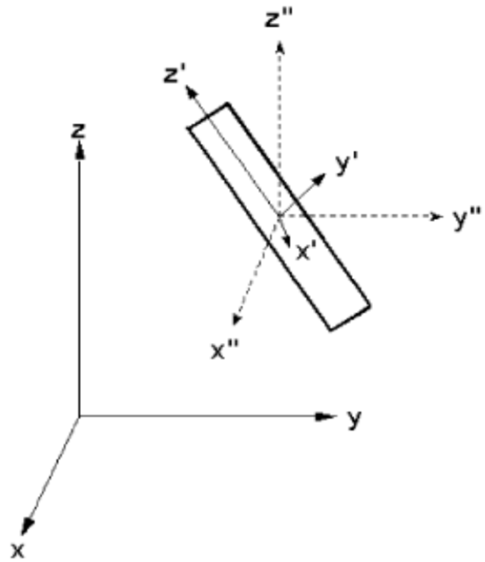
Introduction

- In engineering predictions of dispersed two-phase flow spherical shape for particles is the general assumption.
- Most practical situations particles are irregular or have certain shape (e.g., granulates or fibres)
- Dynamics of non-spherical particles are substantially different from that of spherical (i.e., pitching and rotational torques)
- Analytical expressions of forces & moments are known for non-spherical regular particles in Stokes regime ($Re_p << 1$)
- For moderate Re_p, not much information available
- In such case, fully resolved DNS can be used to extract flow coefficients depending on Re_p and particle shape (e.g., Hölzer & Sommerfeld, 2009; Vakil & Green, 2009; Zastawny et al., 2012)
- Final goal: develop an engineering tool to predict dispersed two-phase flows laden with non-spherical particles

Numerical approximation

- System of interest: behaviour of non-spherical particles immersed in turbulent channel flow
- System described by the Euler-Lagrange approach
- Particles considered as point masses with dynamics given by linear and angular (orientation) momentum equations
- Fluid field computed by RANS (Reynolds Stress Model), modified by the presence of particles (two-way coupling)
- Non-spherical particles motion due to drag and lift forces, whose coefficients were previously obtained by DNS, as well as pitching and rotational torques.
- Here, elongated fibres at intermediate Reynolds numbers are considered. Expressions for the interaction with solid walls have been developed.
- Outputs: particle mean velocity, fluctuating velocity components (stream-wise and vertical) and concentration profiles in the channel.

Non-spherical particles governing equations



$$\overrightarrow{x'} = A \cdot \overrightarrow{x''}$$

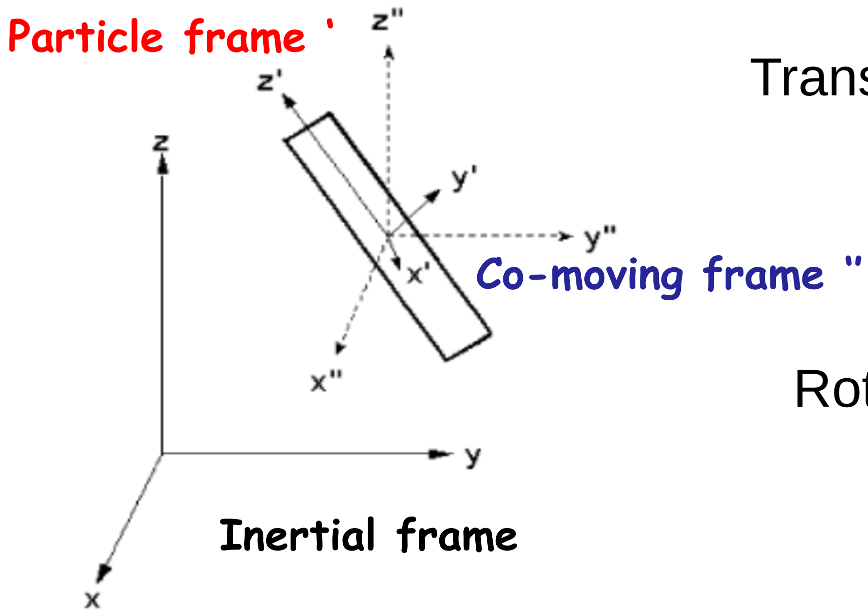
Rotation matrix **A** written in terms of Euler parameters

$$\mathbf{A} = \begin{bmatrix} 1-2\left(\varepsilon_2^2+\varepsilon_3^2\right) & 2\left(\varepsilon_1\varepsilon_2+\varepsilon_3\eta\right) & 2\left(\varepsilon_1\varepsilon_3-\varepsilon_2\eta\right) \\ 2\left(\varepsilon_1\varepsilon_2-\varepsilon_3\eta\right) & 1-2\left(\varepsilon_1^2+\varepsilon_3^2\right) & 2\left(\varepsilon_3\varepsilon_2+\varepsilon_1\eta\right) \\ 2\left(\varepsilon_1\varepsilon_3+\varepsilon_2\eta\right) & 2\left(\varepsilon_3\varepsilon_2-\varepsilon_1\eta\right) & 1-2\left(\varepsilon_2^2+\varepsilon_1^2\right) \end{bmatrix}$$

Time evolution of Euler parameters

$$\begin{bmatrix} \dfrac{d\varepsilon_1}{dt} \\ \dfrac{d\varepsilon_2}{dt} \\ \dfrac{d\varepsilon_3}{dt} \\ \dfrac{d\eta}{dt} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \eta\omega_{x'} - \varepsilon_3\omega_{y'} + \varepsilon_2\omega_{z'} \\ \varepsilon_3\omega_{x'} + \eta\omega_{y'} - \varepsilon_1\omega_{z'} \\ -\varepsilon_2\omega_{x'} + \varepsilon_1\omega_{y'} + \eta\omega_{z'} \\ -\varepsilon_1\omega_{x'} - \varepsilon_2\omega_{y'} - \varepsilon_3\omega_{z'} \end{bmatrix}$$

Non-spherical particles governing equations



Translational motion (inertial frame)

$$m_p \frac{d\vec{u}_p}{dt} = \vec{F}$$

Rotational motion (particle frame)

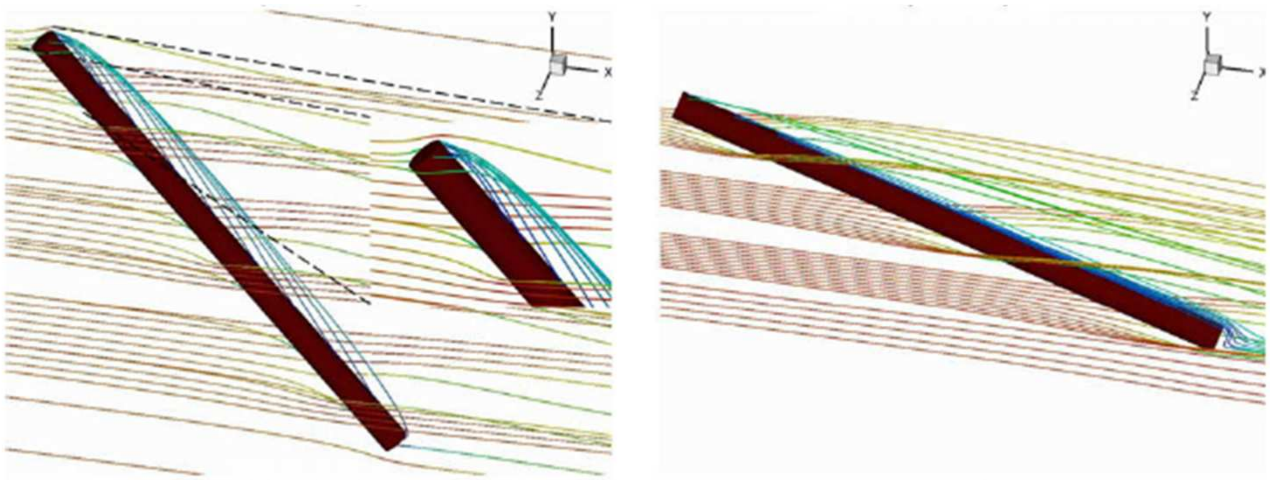
$$I_{x'} \frac{d\omega_{x'}}{dt} - \omega_{y'}\omega_{z'}\left(I_{y'} - I_{z'}\right) = T_{x'}$$
$$I_{y'} \frac{d\omega_{y'}}{dt} - \omega_{x'}\omega_{z'}\left(I_{z'} - I_{x'}\right) = T_{z'}$$
$$I_{z'} \frac{d\omega_{z'}}{dt} - \omega_{y'}\omega_{x'}\left(I_{x'} - I_{y'}\right) = T_{z'}$$

Forces: drag and lift

Torques: pitching and rotational

Flow coefficients cylindrical fibres

Cylinders Vakil & Green, C&F (2009)



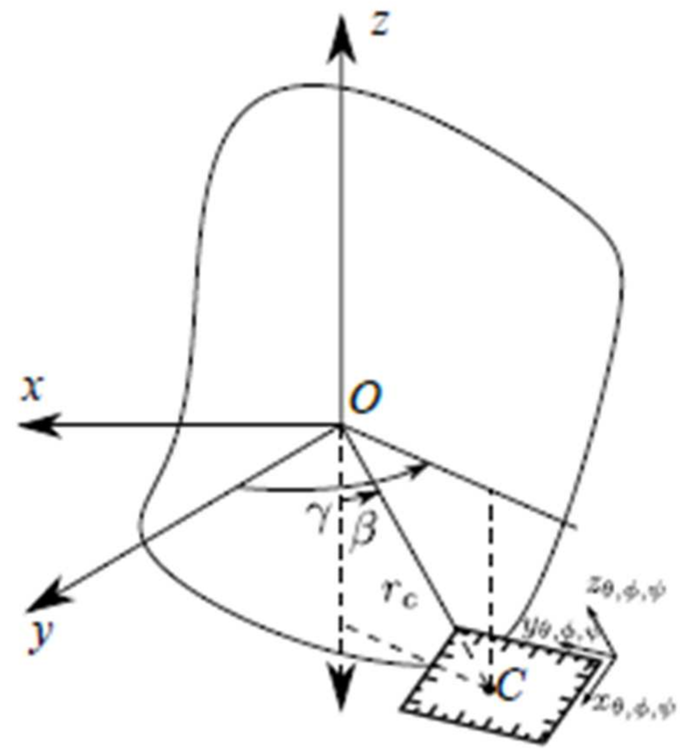
($\alpha = 50^\circ$) ($\alpha = 20^\circ$)

Correlations flow coefficients obtained by DNS depending on particle Reynolds numbers, orientation and aspect ratio (AR)

$$C_{D,cyl} = \frac{F_D}{\frac{1}{2}\rho \tilde{u}^2 LD} \qquad C_{L,cyl} = \frac{F_L}{\frac{1}{2}\rho \tilde{u}^2 LD}$$

Torque coefficients follow the approach of Yin et al., CES (2003)

Non-spherical particle – wall interaction

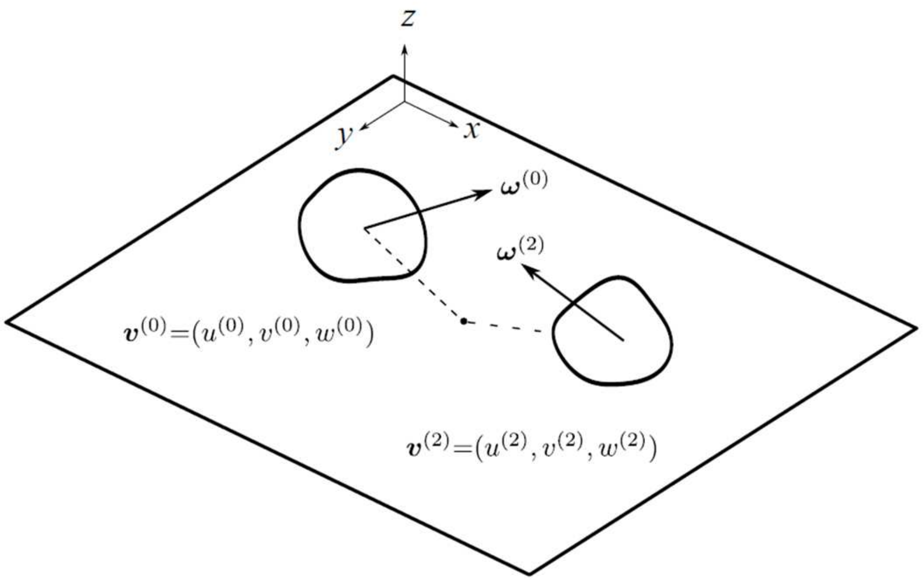


$$C : (x, y, z) \rightarrow (r_c, \beta, \gamma)$$
$$0 \leq \gamma \leq 2\pi; \; 0 \leq \beta \leq \pi$$

- β, γ and r_c depend on shape and orientation of particle
- β, γ and r_c are determined using real values of θ, ϕ and ψ before wall collision and are determined using an analytic model
- A orientation matrix **A** is necessary defined to get the orientation the particle before wall collision

Non-spherical particle – wall interaction

Hard particle approximation



$$m(\mathbf{v}^{(2)} - \mathbf{v}^{(0)}) = \mathbf{J}$$
$$I(\omega^{(2)} - \omega^{(0)}) = -\mathbf{r} \times \mathbf{J}$$

Crowe et al. (2011)

I. Particle stops sliding in compression period
II. Particle stops sliding in recovery period
III. Particle slides throughout compression and recovery period

Additional Assumptions:
Collision is modeled by the coefficient of restitution and friction is modeled by Coulomb's law

Non-spherical particle – wall interaction

Case I

Linear velocities

$$u^{(2)} = \omega_y^{(2)} r_c \cos\beta - \omega_z^{(2)} r_c \sin\beta \sin\gamma$$

$$v^{(2)} = \omega_z^{(2)} r_c \sin\beta \cos\gamma - \omega_x^{(2)} r_c \cos\beta$$

$$w^{(2)} = w^{(0)} + (1+e) J_z^{(1)} / m_p$$

Angular velocities

$$\omega_x^{(2)} = \frac{I_x \omega_x^{(0)} - m_p v^{(0)} r_c \cos\beta - (1+e) r_c \sin\beta \sin\gamma J_z^{(1)} + m_p \omega_z^{(2)} r_c^2 \sin\beta \cos\beta \cos\gamma}{I_x + m_p r_c^2 \cos^2\beta}$$

$$\omega_y^{(2)} = \frac{I_y \omega_y^{(0)} + m_p u^{(0)} r_c \cos\beta + (1+e) r_c \sin\beta \cos\gamma J_z^{(1)} + m_p \omega_z^{(2)} r_c^2 \sin\beta \cos\beta \sin\gamma}{I_y + m_p r_c^2 \cos^2\beta}$$

$$\omega_z^{(2)} = \frac{I_z \omega_z^{(0)} + m_p v^{(0)} r_c \sin\beta \cos\gamma - m_p u^{(0)} r_c \sin\beta \sin\gamma + m_p \omega_y^{(2)} r_c^2 \sin\beta \cos\beta \sin\gamma - m_p \omega_x^{(2)} r_c^2 \sin\beta \cos\beta \cos\gamma}{I_z + m_p r_c^2 \sin^2\beta}$$

Linear impulse normal to the wall

$$J_z^{(1)} = m_p (\omega_x^{(1)} r_c \sin\beta \sin\gamma - \omega_y^{(1)} r_c \sin\beta \cos\gamma - w_0)$$

$$\omega_y^{(1)} = \frac{m_p I_y I_{22} \omega_y^{(0)} + m_p I_x I_{12} \omega_x^{(0)} + m_p I_z I_{31} \omega_z^{(0)} + m_p r_c \cos\beta u_0 I_1^2 - m_p r_c \sin\beta \cos\gamma v_0 I_2^2 - m_p r_c \sin\beta \cos\gamma w_0 I_3^2}{m_p I_y r_c^2 \sin^2\beta \sin^2\gamma (I_x + I_z + m_p r_c^2) + m_p I_x r_c^2 \cos^2\beta \sin^2\gamma (I_y + I_z + m_p r_c^2) + m_p I_y r_c^2 \cos^2\beta (I_x + I_y + m_p r_c^2) + I_x I_y I_z}$$

$$\omega_x^{(1)} = \frac{m_p I_x I_{11} \omega_x^{(0)} + m_p I_y I_{23} \omega_y^{(0)} + m_p I_z I_{32} \omega_z^{(0)} + m_p r_c \cos\beta u_0 I_4^2 - m_p r_c \sin\beta \cos\gamma v_0 I_5^2 + m_p r_c \sin\beta \cos\gamma w_0 I_6^2}{m_p I_y r_c^2 \sin^2\beta \sin^2\gamma (I_x + I_z + m_p r_c^2) + m_p I_x r_c^2 \cos^2\beta \sin^2\gamma (I_y + I_z + m_p r_c^2) + m_p I_y r_c^2 \cos^2\beta (I_x + I_y + m_p r_c^2) + I_x I_y I_z}$$

Non-spherical particle – wall interaction

Case I

Extra relations

$$I_1^2 = m_p r_c^2 \sin^2\beta \cos^2\gamma\, I_x + m_p r_c^2 \sin^2\beta \sin^2\gamma\, I_z + m_p r_c^2 \cos^2\beta\, I_z + I_x I_z$$

$$I_2^2 = m_p r_c^2 \sin\beta \sin\gamma \cos\beta\, I_z - m_p r_c^2 \sin\beta \sin\gamma \cos\beta\, I_x$$

$$I_3^2 = m_p r_c^2 \sin^2\beta \cos^2\gamma\, I_x + m_p r_c^2 \sin^2\beta \sin^2\gamma\, I_x + m_p r_c^2 \cos^2\beta\, I_z + I_y I_z$$

$$I_4^2 = m_p r_c^2 \sin\beta \cos\gamma \cos\beta\, I_z - m_p r_c^2 \sin\beta \cos\gamma \cos\beta\, I_x$$

$$I_5^2 = m_p r_c^2 \sin^2\beta \cos^2\gamma\, I_z + m_p r_c^2 \sin^2\beta \sin^2\gamma\, I_y + m_p r_c^2 \cos^2\beta\, I_z + I_y I_z$$

$$I_6^2 = m_p r_c^2 \sin^2\beta \cos^2\gamma\, I_y + m_p r_c^2 \sin^2\beta \sin^2\gamma\, I_y + m_p r_c^2 \cos^2\beta\, I_z + I_y I_z$$

$$I_{11} = m_p r_c^4 \sin^2\beta \cos^2\gamma + r_c^2 \cos^2\beta\, I_z + r_c^2 \sin^2\beta \sin^2\gamma\, I_y + r_c^2 \sin^2\beta \cos^2\gamma (I_y + I_z) + I_y I_z / m_p$$

$$I_{22} = m_p r_c^4 \sin^2\beta \sin^2\gamma + r_c^2 \cos^2\beta\, I_z + r_c^2 \sin^2\beta \cos^2\gamma\, I_x + r_c^2 \sin^2\beta \cos^2\gamma (I_x + I_z) + I_x I_z / m_p$$

$$I_{12} = I_{23} = m_p r_c^2 \sin^2\beta \sin\gamma \cos\gamma + r_c^2 \sin^2\beta \sin\gamma \cos\gamma\, I_z$$

$$I_{31} = m_p r_c^2 \sin\beta \sin\gamma \cos\beta + r_c^2 \sin\beta \sin\gamma \cos\beta\, I_x$$

$$I_{32} = m_p r_c^2 \sin\beta \cos\gamma \cos\beta + r_c^2 \sin\beta \cos\gamma \cos\beta\, I_x$$

Non-spherical particle – wall interaction

Case II

Linear velocities

$$u^{(2)} = \omega_y^{(2)} r_c \cos\beta - \omega_z^{(2)} r_c \sin\beta \sin\gamma$$

$$v^{(2)} = \omega_z^{(2)} r_c \sin\beta \cos\gamma - \omega_x^{(2)} r_c \cos\beta$$

$$w^{(2)} = w^{(0)} + (1+e) J_z^{(1)} / m_p$$

Angular velocities

$$\omega_x^{(2)} = \frac{I_x \omega_x^{(0)} - m_p v^{(0)} r_c \cos\beta - (1+e) r_c \sin\beta \sin\gamma J_z^{(1)} + m_p \omega_z^{(2)} r_c^2 \sin\beta \cos\beta \cos\gamma}{I_x + m_p r_c^2 \cos^2\beta}$$

$$\omega_y^{(2)} = \frac{I_y \omega_y^{(0)} + m_p u^{(0)} r_c \cos\beta + (1+e) r_c \sin\beta \cos\gamma J_z^{(1)} + m_p \omega_z^{(2)} r_c^2 \sin\beta \cos\beta \sin\gamma}{I_y + m_p r_c^2 \cos^2\beta}$$

$$\omega_z^{(2)} = \frac{I_z \omega_z^{(0)} + m_p v^{(0)} r_c \sin\beta \cos\gamma - m_p u^{(0)} r_c \sin\beta \sin\gamma + m_p \omega_y^{(2)} r_c^2 \sin\beta \cos\beta \sin\gamma - m_p \omega_x^{(2)} r_c^2 \sin\beta \cos\beta \cos\gamma}{I_z + m_p r_c^2 \sin^2\beta}$$

Linear impulse normal to the wall

$$J_z^{(1)} = \frac{m_p I_x I_y (\omega_x^{(0)} r_c \sin\beta \sin\gamma - \omega_y^{(0)} r_c \sin\beta \cos\gamma - w^{(0)})}{I_x I_y + m_p r_c^2 [I_x (\varepsilon_x \mu \sin\beta \cos\beta \cos\gamma + \sin^2\beta \cos^2\gamma) + I_y (\varepsilon_y \mu \sin\beta \cos\beta \sin\gamma + \sin^2\beta \sin^2\gamma)]}$$

Non-spherical particle – wall interaction

Case III

Linear velocities

$$u^{(2)} = u^{(0)} - \varepsilon_x \mu (1+e) \frac{J_z^{(1)}}{m_p}$$

$$v^{(2)} = v^{(0)} - \varepsilon_y \mu (1+e) \frac{J_z^{(1)}}{m_p}$$

$$w^{(2)} = w^{(0)} + (1+e) \frac{J_z^{(1)}}{m_p}$$

Angular velocities

$$\omega_x^{(2)} = \omega_x^{(0)} - (1+e)[\varepsilon_y \mu \cos\beta + \sin\beta \sin\gamma] \frac{J_z^{(1)} r_c}{I_x}$$

$$\omega_y^{(2)} = \omega_y^{(0)} + (1+e)[\varepsilon_x \mu \cos\beta + \sin\beta \cos\gamma] \frac{J_z^{(1)} r_c}{I_y}$$

$$\omega_z^{(2)} = \omega_z^{(0)} + (1+e)[\varepsilon_y \mu \cos\gamma - \varepsilon_x \mu \sin\gamma] \sin\beta \frac{J_z^{(1)} r_c}{I_z}$$

Linear impulse normal to the wall

$$J_z^{(1)} = \frac{m_p I_x I_y (\omega_x^{(0)} r_c \sin\beta \sin\gamma - \omega_y^{(0)} r_c \sin\beta \cos\gamma - w^{(0)})}{I_x I_y + m_p r_c^2 [I_x (\varepsilon_x \mu \sin\beta \cos\beta \cos\gamma + \sin^2\beta \cos^2\gamma) + I_y (\varepsilon_y \mu \sin\beta \cos\beta \sin\gamma + \sin^2\beta \sin^2\gamma)]}$$

Non-spherical particle – wall interaction

Conditions for occurrence of the cases

Case I & II

$$\frac{w^{(0)}}{u^{(0)} - \omega_y^{(0)} r_c \cos\beta + \omega_z^{(0)} r_c \sin\beta \sin\gamma} < -\frac{1}{(1 + m_p r_c^2 \cos^2\beta / I_y)\mu(e+1)}$$

Case III

$$-\frac{1}{(1 + m_p r_c^2 \cos^2\beta / I_y)\mu(e+1)} < \frac{w^{(0)}}{u^{(0)} - \omega_y^{(0)} r_c \cos\beta + \omega_z^{(0)} r_c \sin\beta \sin\gamma} < 0$$

Above equations reduce in the 2D case to those of Sommerfeld (2002) and in the case of spherical particles to those of Crowe, Sommerfeld, Tsuji (1998).

Non-spherical particle – wall interaction

Case IV, ideal collision (e = 1, μ = 0)

Linear velocities

$$u^{(2)} = u^{(0)}$$

$$v^{(2)} = v^{(0)}$$

$$w^{(2)} = w^{(0)} + \frac{2J_z^{(1)}}{m_p}$$

Angular velocities

$$\omega_x^{(2)} = \omega_x^{(0)} - 2\sin\beta\sin\gamma\,\frac{J_z^{(1)} r_c}{I_x}$$

$$\omega_y^{(2)} = \omega_y^{(0)} + 2\sin\beta\cos\gamma\,\frac{J_z^{(1)} r_c}{I_y}$$

$$\omega_z^{(2)} = \omega_z^{(0)}$$

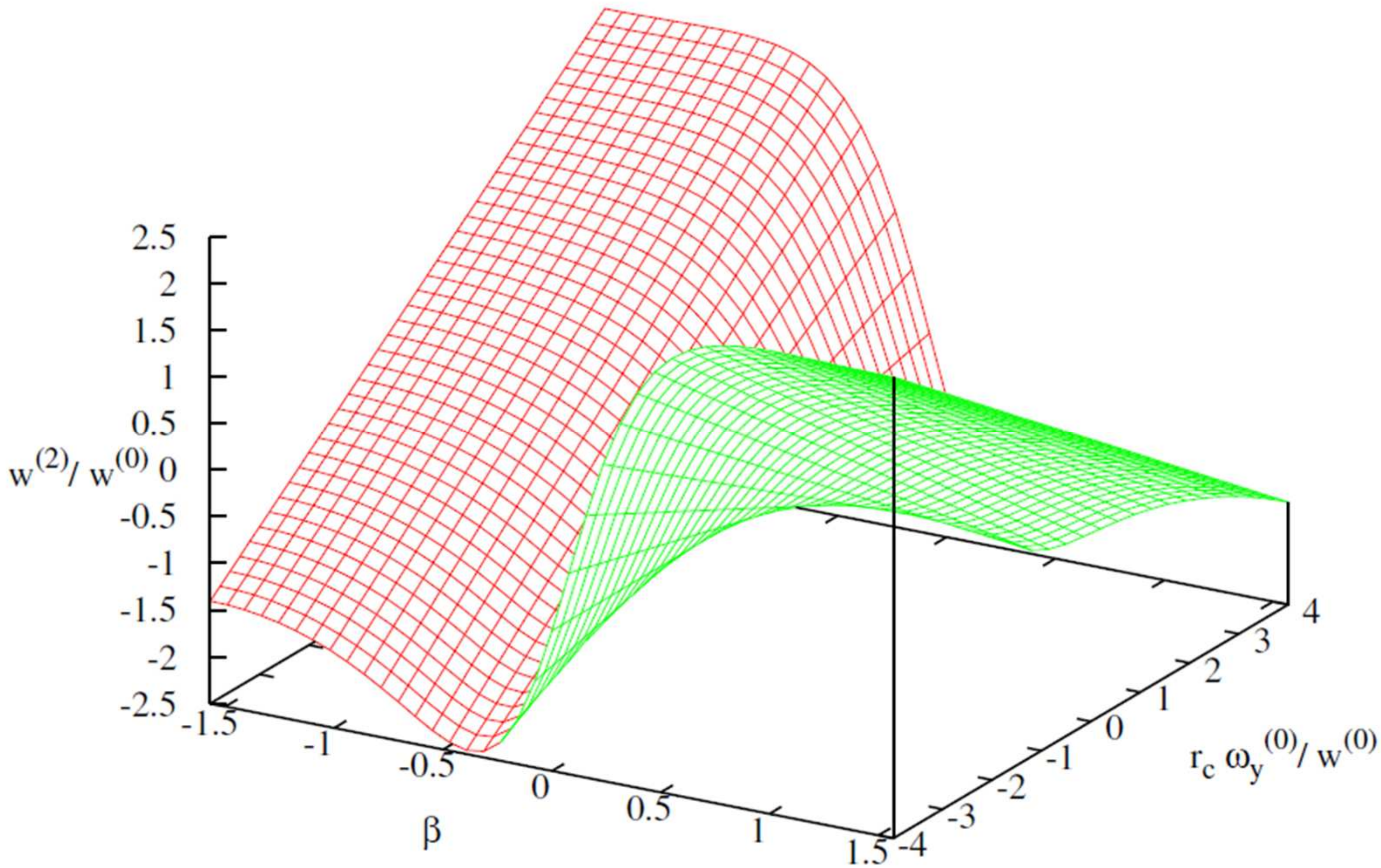
Linear impulse normal to the wall

$$J_z^{(1)} = \frac{m_p I_x I_y(\omega_x^{(0)} r_c \sin\beta\sin\gamma - \omega_y^{(0)} r_c \sin\beta\cos\gamma - w^{(0)})}{I_x I_y + m_p r_c^2 \sin^2\beta(I_x\cos^2\gamma + I_y\sin^2\gamma)}$$

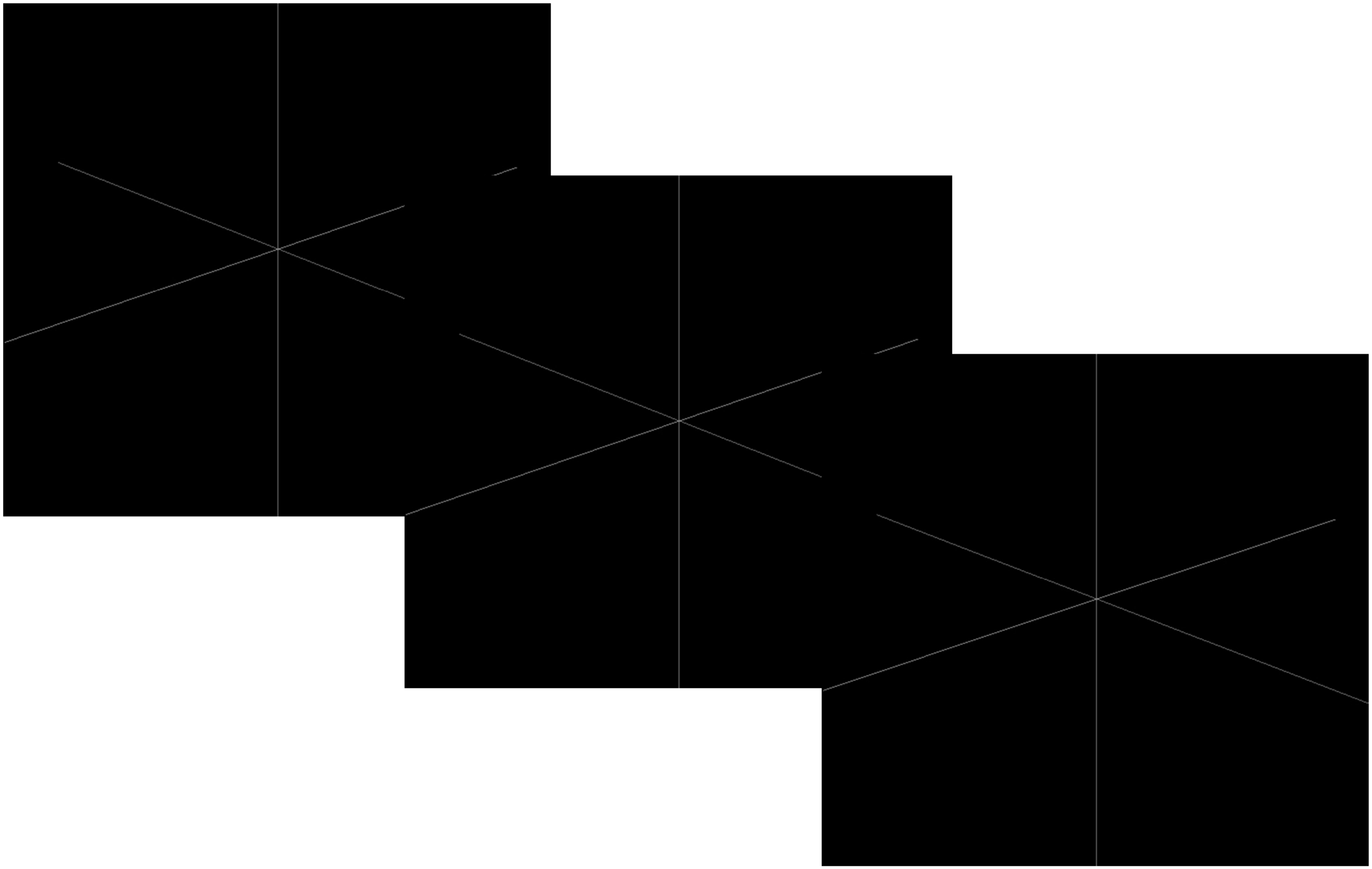
Non-spherical particle – wall interaction

Simplified 2D equation cylinders (ideal case)

$$\frac{w^{(2)}}{w^{(0)}} = 1 - 2\left(\frac{1 + r_c \omega_y^{(0)} \sin\beta / w^{(0)}}{1 + (r_c/k_y)^2 \sin^2\beta}\right)$$



Fibre – wall interaction visualisation

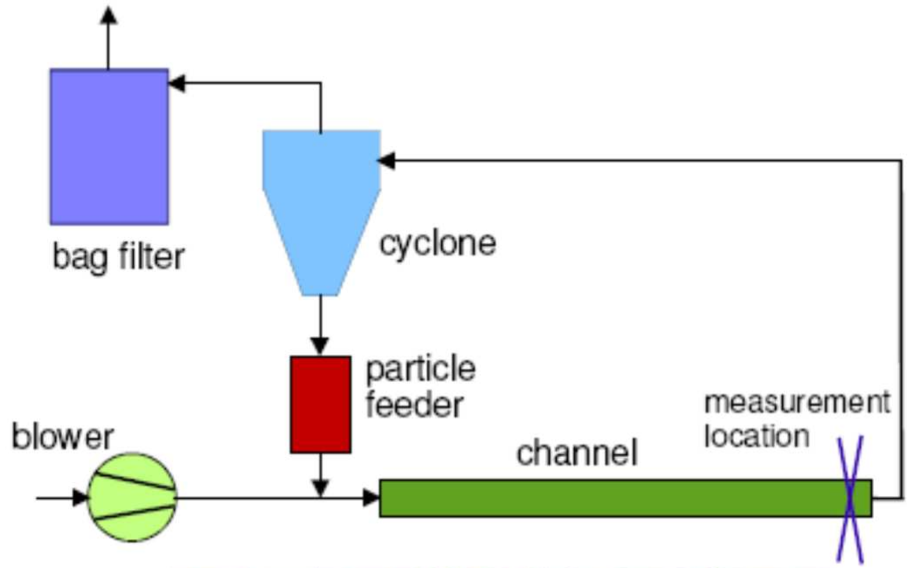


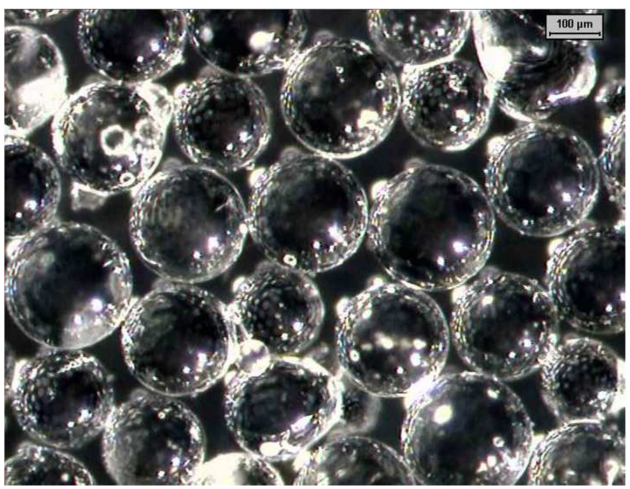
Experimental rig

Channel Flow Configuration

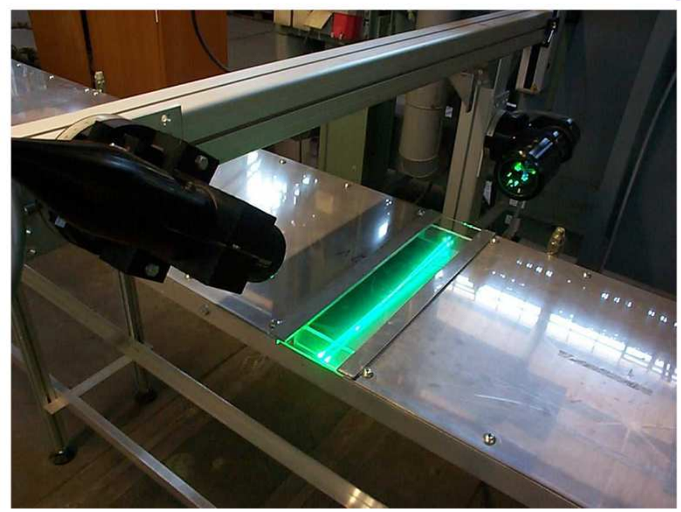
Experimental studies by Kussin (2004)

Horizontal Channel:
Length: 6 m
Width: 350 mm
Height: 35 mm
U_{av} = 20 m/s
Measuring section: 5.8 m

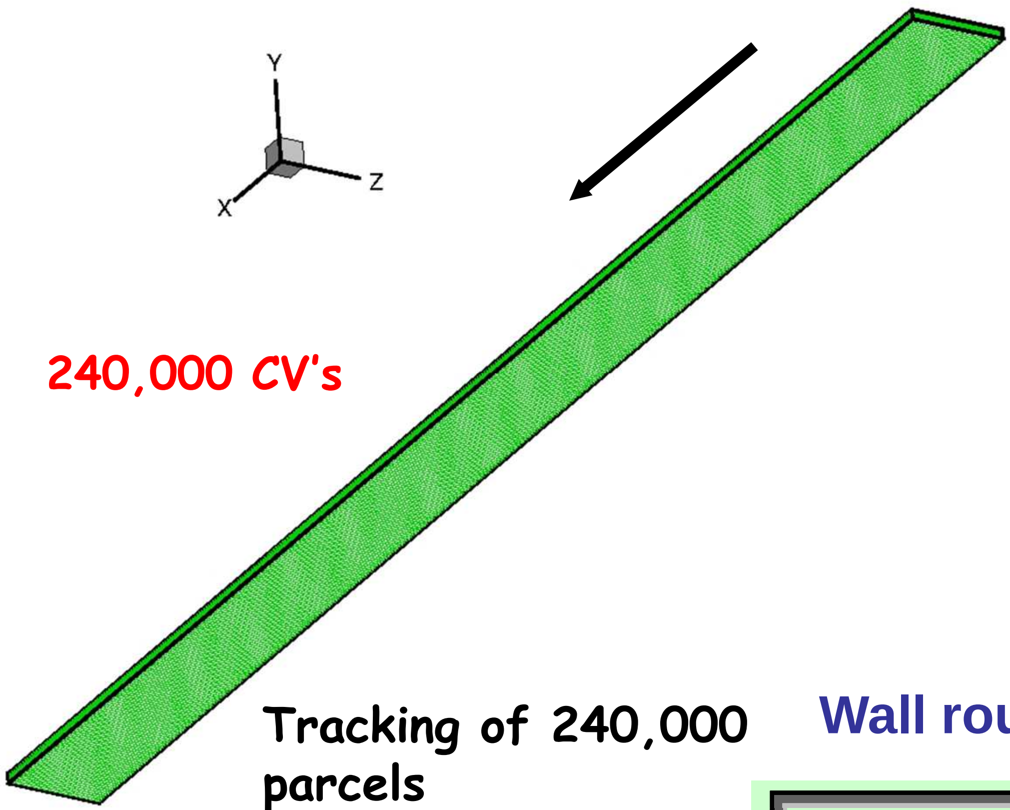




Spherical glass beads, ρ_p = 2450 kg/m^3



Numerical set-up



240,000 CV's

Tracking of 240,000 parcels

Horizontal Channel:

6 m x 350 mm x 35 mm

U_{av} = 20 m/s

Measuring section: 5.8 m

ρ = 1.25 kg/m^3

$\mu = 1.8 \cdot 10^{-5}$ N·s/m^2

ρ_p = 2450 kg/m^3

D_p = 130 μm

η = 0.1

Tracked particles

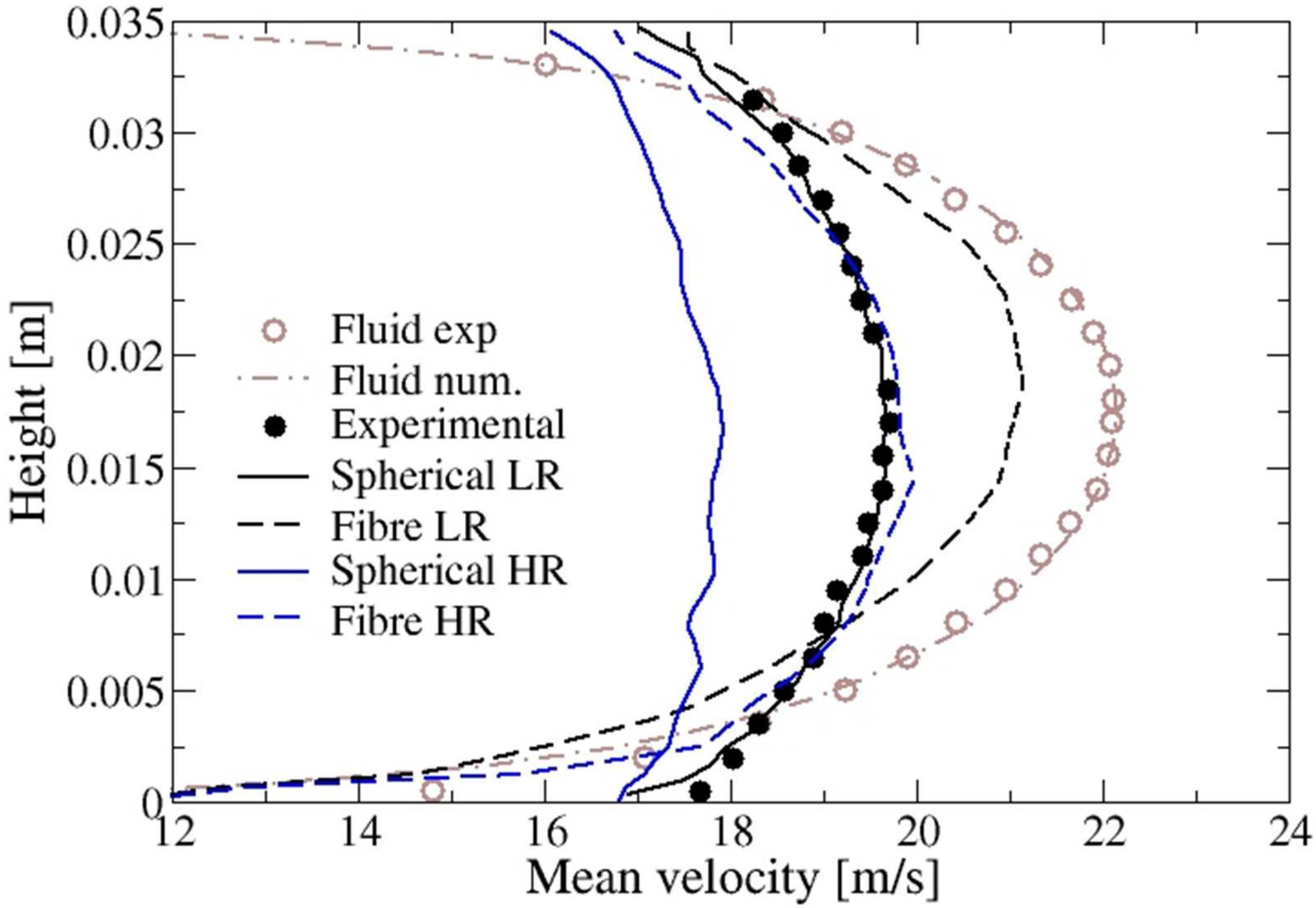
Spheres

Fibres (aspect ratio 13)

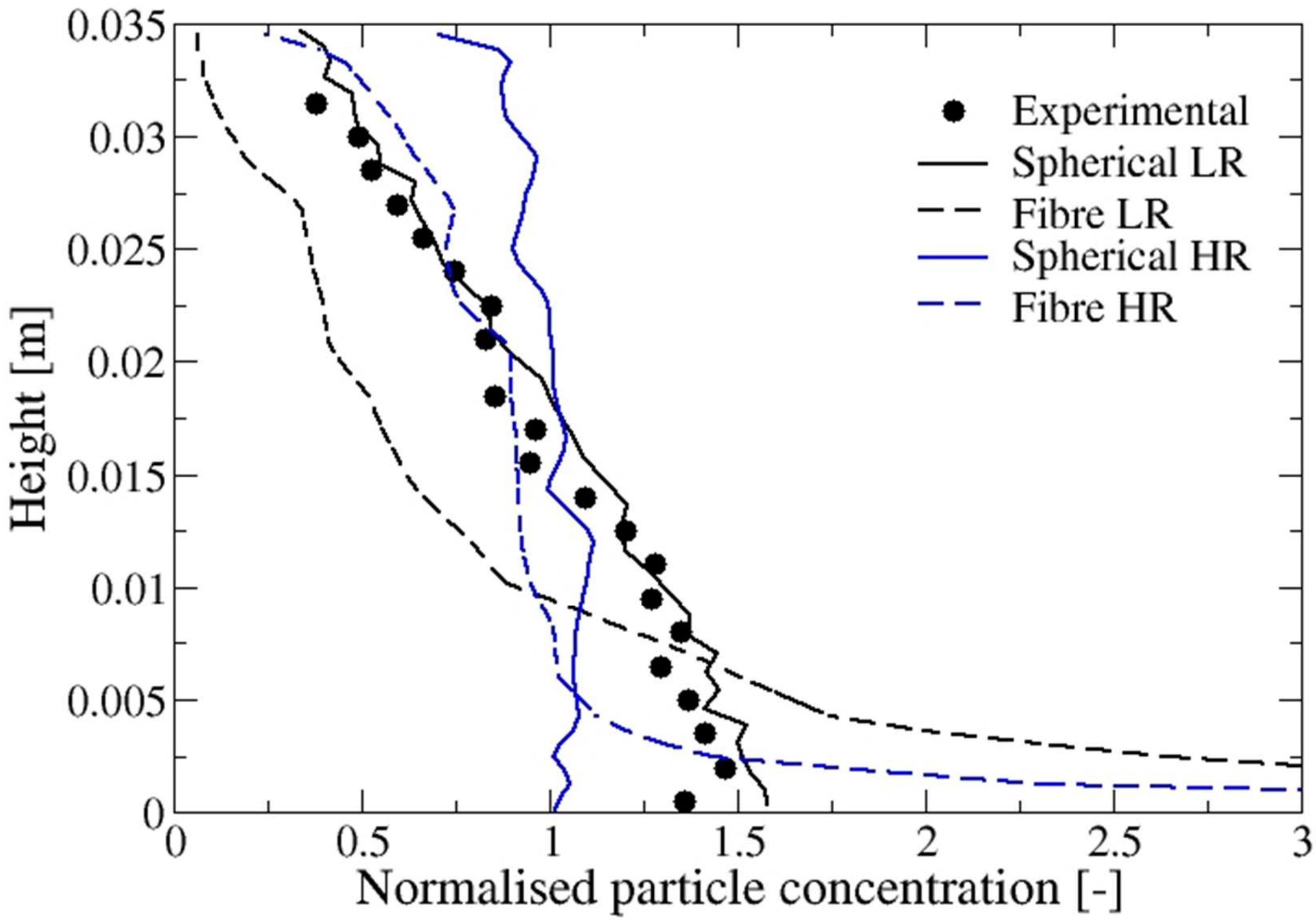
Wall roughness (stochastic model)

Degree of roughness	Mean roughness in stream-wise direction [μm]	Mean roughness in lateral direction [μm]
R0 (LR)	2.32	2.09
R2 (HR)	6.83	6.89

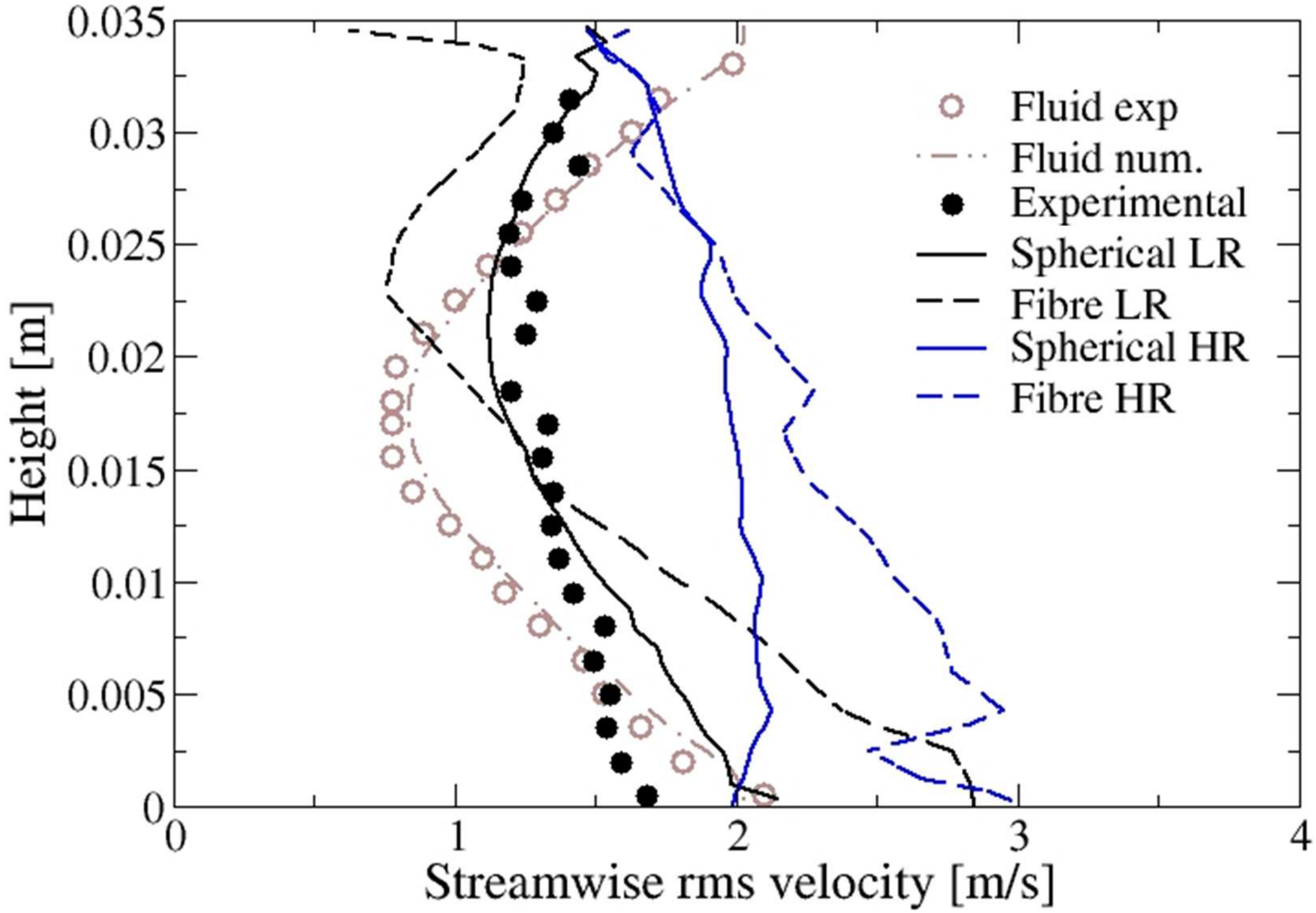
Results cylindrical particles



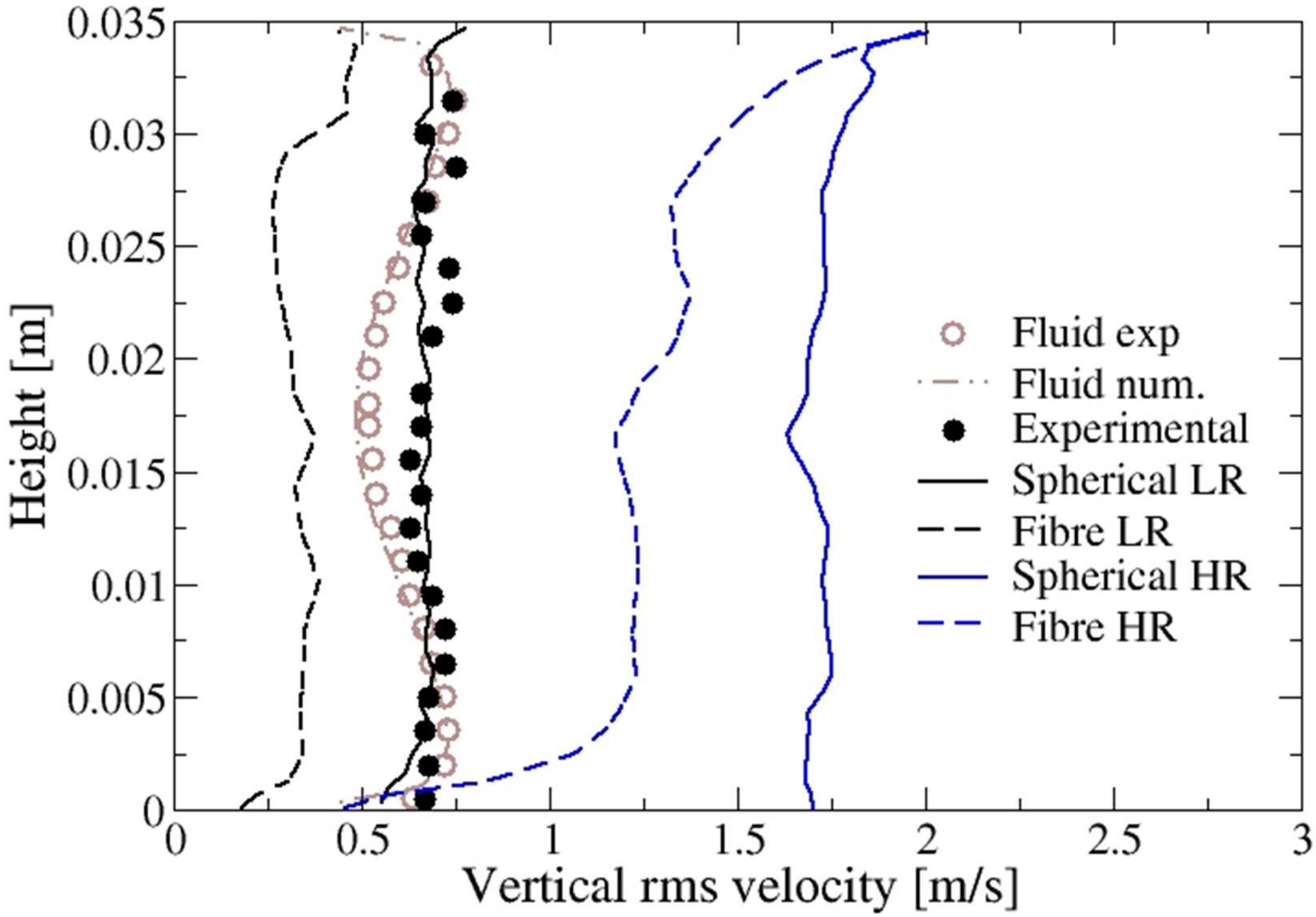
Results cylindrical particles



Results cylindrical particles



Results cylindrical particles



Conclusions

- The Euler/Lagrange approach has been used to investigate the transport of spheres and elongated fibres in a turbulent channel flow
- Equations for the interaction of general non-spherical particles with walls have been presented in the context of "hard particles"
- Depending of angle of impact and rotational velocity one or two collisions with the wall can be observed.
- As expected, fibres follow better than spheres the fluid flow, as they tend to be aligned perpendicular to the relative velocity, maximizing the drag
- From the concentration profiles, fibres show more gravitational settling than spheres of the same size
- Streamwise rms velocity of fibres is more asymmetric than that of spheres
- Vertical rms velocity is remarkably lower in the case of elongated cylinders than for spherical particles, as happens with other type of irregular shapes